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Cecilia Bellora, Jean-Marc Bourgeon. Agricultural Trade, Biodiversity Effects and Food Price Volatility. 2014. hal-00969083

**HAL Id: hal-00969083**

**<https://hal.science/hal-00969083>**

Preprint submitted on 2 Apr 2014

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ÉCOLE POLYTECHNIQUE

CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE



## AGRICULTURAL TRADE, BIODIVERSITY EFFECTS AND FOOD PRICE VOLATILITY

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Cahier n° 2014-09

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# Agricultural Trade, Biodiversity Effects and Food Price Volatility

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April 1, 2014

## Abstract

We examine the importance of production risks in agriculture due to biotic elements such as pests in determining the pattern of trade and the distribution of prices in a Ricardian two-country setup. These elements create biodiversity effects that result in an incomplete specialization at the free trade equilibrium. Their influence on idiosyncratic production risks evolves depending on the countries' openness to trade. Pesticides allow these effects to diminish but they are damaging for the environment and human health. When regulating farming practices, governments have to counterbalance these side-effects with the competitiveness of their agricultural sector on international markets. Nevertheless, restrictions on pesticides under free trade are generally more stringent than under autarky.

**Keywords:** agricultural trade, food prices, agrobiodiversity, pesticides.

**JEL Classification Numbers:** F18, Q17.

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# 1 Introduction

World prices of major agricultural commodities are subject to busts and booms that regularly trigger worries about the functioning of world markets and raise questions about the mechanisms underlying the trend of food prices and their volatility.<sup>1</sup> Price variations are not specific to agricultural markets but it is believed that the volatility of agricultural prices is higher than that of manufactures (Jacks et al., 2011). Various causes for price fluctuations have been proposed, but economic models usually neglect stochastic elements that may affect production processes.<sup>2</sup> While this is probably innocuous for industrial goods, which are rarely subject to hazardous conditions, it is more questionable regarding agricultural products. Indeed, agricultural goods face a large range of potential threats that may severely undercut farms' production. Two types of factors may interfere with the agricultural production: Biotic factors, called pests, which are harmful organisms, either animal pests (insects, rodents, birds, worms, etc ), pathogens (virus, bacteria, fungi, etc), or weeds, and abiotic factors such as water stress, temperature, irradiance and nutrient supply that are often related to weather conditions (Oerke, 2006). Many studies show that crop losses due to pests are critical.<sup>3</sup>

Our analysis highlights the importance of the stochastic elements affecting agricultural production in determining the pattern of food trade, and particularly the biotic ones that relate the biodiversity of cultivated crops to their productivity and create what are dubbed *biodiversity effects* in the following. Because farmland has specificities that differ from one country to the other, trade must result in specialized international crop productions because

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<sup>1</sup>One recent example of public concern was given during the G-20 leaders summit meeting of November 2010: The participants requested several international institutions “to develop options on how to better mitigate and manage the risks associated with the price volatility of food and other agriculture commodities, without distorting market behavior, ultimately to protect the most vulnerable.”

<sup>2</sup>See Gilbert & Morgan (2010) and Wright (2011) for overviews on food price volatility and examinations of the causes of the recent spikes.

<sup>3</sup>Oerke (2006) estimates that the potential losses due to pests (i.e., without crop protection of any kind) between 2001 and 2003 for wheat, rice, maize, soybeans and cotton were respectively 49.8%, 77%, 68.5%, 60% and 80% of the potential yield. Savary et al. (2000) also finds significant losses: combined average level of injuries led to a decrease of 37.2% in rice yield (if injuries were considered individually, the cumulative yield loss would have been 63.4%). Fernandez-Cornejo et al. (1998) compile similar data on previous periods and find that expected losses relative to current or potential yield from insects and diseases range between 2 to 26% and that losses from weeds range between 0% and 50%.

of the implied competitive advantages. In this Ricardian setup, opening to trade should not help much to smooth out production risks since the world is exposed to the idiosyncratic risk of the only country producing each crop. We show that biodiversity effects create a production externality that impedes productivity and results in an incomplete specialization at equilibrium under free trade. Hence, it is not necessary to introduce exogenous frictions in the Ricardian setup to explain that a whole range of crops are produced by several countries. Moreover, because of these productivity losses and imperfect specialization, biotic factors have an impact on the distribution of international food prices that differs from one crop to the other.

To reduce the productive loss due to pests, farmers have recourse to agrochemicals (pesticides, fungicides, herbicides and the like),<sup>4</sup> but these products also have damaging side-effects on the environment and human health that spur governmental intervention.<sup>5</sup>

However, restrictions on the use of pesticides may impede the competitiveness of agriculture on international markets. Governments, when designing these regulations, thus face a tradeoff: either they contain the externality on consumers and tolerate a high level of negative production externality, which diminishes the competitiveness of their agricultural sector, or they are more permissive in the utilization of pesticides to enhance the productivity of their agricultural sector to the detriment of their consumers. We show that when governments use these regulations strategically to enhance the comparative advantages of

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<sup>4</sup>Practices controlling biotic factors include physical (mechanical weeding or cultivation...), biological (crop rotations, cultivar choice, predators...) and chemical measures. They allow a reduction of losses, but do not totally avoid them. In the case of wheat, they reduce potential losses of 50% to average actual losses of about 29% (from 14% in Northwest Europe up to 35% and more in Central Africa and Southeast Asia). Actual losses for soybean average around 26%. The order of magnitude of losses for maize and rice is greater, with actual losses of respectively 40% and 37% (Oerke, 2006).

<sup>5</sup>Pimentel (2005) reports more than 26 million cases worldwide of non-fatal pesticide poisoning and approximately 220 000 fatalities. He estimates that the effects of pesticides on human health cost about 1.2 billion US\$ per year in the United States. Mammals and birds are also affected. Farmland bird population decreased by 25% in France between 1989 and 2009 (Jiguet et al., 2012), and a sharp decline was also observed in the whole EU during the same period (EEA, 2010). Pesticides also contaminate water and soils and significantly affect water species both locally and regionally (Beketov et al., 2013). Many countries have adopted regulations that forbid the most harmful molecules, set provisions on the use and storage of pesticides, and intend to promote their sustainable use. For example, only a fourth of the previously marketed molecules passed the safety assessment made within the European review process ended in 2009 (EC, 2009). The use of pesticides seems to follow a decreasing trend: in Europe, quantities sold decreased by 24% between 2001 and 2010 (ECP, 2013).

their agriculture, trade results in an environmental “dumping” situation where regulations are more lenient than they otherwise would be. Nevertheless, restrictions on pesticides under free trade are more stringent than under autarky.

To investigate the importance biodiversity effects on the pattern of trade and on price volatility, we re-examine the standard Ricardian model of trade as developed by Dornbusch et al. (DFS, 1977), which considers the impact of trade on two economies that produce a continuum of goods. The productive efficiencies of these economies determine their comparative advantages, resulting in a division of the production into two specific ranges of products, one for each country. Here, we consider two-sector economies, an industrial/service sector that produces a homogeneous good with equal productivity in the two countries (which serves as the numeraire), and an agricultural sector that produces a range of goods with different potential yields, the effective crop yields depending on biotic and abiotic factors. Many agro-ecology studies show that genetic diversity within and among crop species improves the resilience of agricultural ecosystems to adverse climatic conditions and harmful organisms. Crop biodiversity reduction, observed in many modern agricultural systems, increases the vulnerability of crops to these stresses.<sup>6</sup> To account for these relationships in a tractable way, we follow Weitzman (2000): the larger the share of farmland dedicated to a crop, the more its parasitic species proliferate, and thus the more fields of that crop are at risk of being wiped out.<sup>7</sup> More specifically, farming a given plot of land is a Bernoulli trial in which the probability of success depends on the way land is farmed in the country: the larger the share of farmland devoted to a particular crop, the higher the risk that a field of that crop is destroyed by biotic or abiotic factors. This translates at the country level by crop yields that follow binomial distributions parameterized by farming intensities. Pesticides have a

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<sup>6</sup>This is demonstrated in Tilman et al. (2005) using simple ecological models that describe the positive influence of diversity on the biomass produced by both natural and managed (i.e. agricultural) ecosystems, and corroborated by empirical results detailed in Tilman & Downing (1994) and Tilman et al. (1996). See also Smale et al. (1998), Di Falco & Perrings (2005) or Di Falco & Chavas (2006) for econometric investigations.

<sup>7</sup>Weitzman (2000) makes an analogy between parasite-host relationships and the species-area curve that originally applies to islands: the bigger the size of an island, the more species will be located there. He compares the total biomass of a uniform crop to an island in a sea of other biomass. A large literature in ecology uses the species-area curve which is empirically robust not only for islands but for uniform regions more generally (May, 2000; Garcia Martin & Goldenfeld, 2006; Drakare et al., 2006; Plotkin et al., 2000; Storch et al., 2012).

positive impact at both the individual level (direct effect) and regional level (indirect, cross-externality effect): by destroying pests on her field, a farmer improves the resilience of her plot and reduces the risk borne by the adjacent fields. This cross-externality between fields of the same crop thus attenuates the negative biodiversity effects. We model the regulation on pesticides as an environmental tax on their price whose proceeds are redistributed to farmers (being thus fiscally neutral). The agricultural sector being competitive, the expected revenue of farming at equilibrium is the same whatever the crop being produced, equal to the wage and the rental price of land. We abstract from risk-aversion by assuming that farmers and consumers are risk-neutral.<sup>8</sup> Although by assumption potential crop yields are different (relative yields are described by a continuous function monotonous over the range of crops) consumer preferences and the other characteristics of the two economies we consider are such that the autarky case displays symmetric situations: the agricultural revenue, land rent and environmental tax are at the same levels; only crop prices differ. We then consider the free trade situation and show that biodiversity effects result in an incomplete specialization of the countries. The intuition is as follows: without biodiversity effects, free trade results in the same specialization as described by DFS, a threshold crop delimiting the production range specific to each country. With biodiversity effects, such a clear-cut situation with one threshold crop can no longer exist because the intensification of farming reduces expected crop yields: an increase in acreage devoted to a crop reduces its expected yield and, symmetrically, a diminution of its acreage increases it. As a result, the two countries produce a whole range of crops delimited by two threshold crops, their shares of production evolving according to the relative yields. This range of crops depends on the environmental taxes, which also determine the country's share of the worldwide agricultural revenue. Although we consider competitive economies, we show that the free trade outcome is like the "strategic trade" problem first investigated by Brander & Spencer (1985) (see Helpman & Krugman, 1989, for a detailed analysis), the environmental taxes being strategic substitutes. This is the case with or without biodiversity effects, since pesticides have a direct effect on produc-

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<sup>8</sup>Risk-aversion would reduce specialization under free trade to diversify production risks among countries. Abstracting from this diversification effect allows us to isolate the impact of biodiversity on specialization.

tion, but distinguishing between these two cases allows us to detail the biodiversity effects more thoroughly. Two countervailing effects are at work when accounting for the biodiversity effects: as specialization increases the negative production externality, governments are induced to lower the tax on pesticides, but since the externality limits trade specialization, the effect of the tax on prices concerns a reduced set of crops, which induces governments to increase the tax.

To solve this game when biodiversity effects are at work, and to compare its equilibrium to the autarky case, we focus on a family of relative potential yield functions that allows for symmetric equilibria. We also characterize a fictitious non-strategic situation where the governments are unaware that regulations have an impact on their revenues. Compared to this non-strategic situation, we show that environmental taxes are lower at the equilibrium of the strategic trade game. Hence, to give a competitive advantage to their farmers, the governments are permissive regarding the use of pesticides. That leniency depends on the way the relative potential yield evolves along the range of crops: the more rapidly the productivity differential increases, the tighter the regulation.

However, we also show that environmental taxes are generally above the autarky level. Indeed, under autarky, governments are willing to accept a more intensive use of pesticides because they allow a decrease in the prices of all crops sold in the country. Domestic consumers bear all the costs and the benefits of the agrochemical use. This is no longer true under free trade because of the (partial) specialization of each country. Domestic agricultural production concerns only a reduced range of products, and some of these crops are also produced abroad: prices of these goods are less reactive to the domestic regulation.

Finally, the distribution of crop production allows us to characterize the market price volatility in each case. Approximations of the expected value, of the standard deviation and of confidence intervals for the market prices are derived using the variation coefficient (the ratio of the standard deviation over the expected value) of the crop production. We show that volatility increases with the amount of specialization in production: the more a country specializes in a crop, the larger the variation coefficient of crop production. This is also true for crops produced by both countries: we derive an explicit formula in the symmetric case



which reveals that a diversification effect is at work. Indeed, when both countries devote the same share of farmland to the same crop (which occurs only for the one crop with the same expected potential yield for both countries) this coefficient is minimum. The farther away from this value, the more specialized and volatile production becomes.

Models of food trade incorporating production shocks generally assume that the distributions of idiosyncratic shocks are not affected by trade policies and that free trade allows the diversification of risk. This is the case e.g. in Newbery & Stiglitz (1984) which shows that when financial markets are missing, production choices of risk-averse farmers under free trade are different from those made under autarky and may result in welfare losses. These models assume that the potential productivity of farmland is the same whatever the country. We take the opposite viewpoint and adopt the Ricardian setup of DFS to account for farmland discrepancies between countries. This setup allows us to describe how idiosyncratic shocks evolve with trade. Recent Ricardian models of trade involving more than two countries (Eaton & Kortum, 2002; Costinot & Donaldson, 2012) incorporate a stochastic component together with trade costs (as in DFS) to specify the pattern of trade and avoid complete specialization. These components are not related to the production process (they are “exogenous”) and of no use in explaining price volatility. Our setup offers such features endogenously: biotic and abiotic factors affect stochastically production which generates price volatility and causes productivity losses that prevent complete specialization at equilibrium.

The rest of this article is organized as follows: the next section details how we integrate biodiversity effects in the agricultural production process. We derive the autarky situation and show that biodiversity effects result in an incomplete specialization under free trade. Section 3 is devoted to the tax policy under free trade. Strategic and non strategic tax policies are derived to analyze the impact of the biodiversity effects on public policy. Their implications on price volatility are exposed in section 4. We discuss the impacts of trade on fertilizer use in section 5. The last section concludes.

## 2 The model

Consider two countries (Home and Foreign) whose economies are composed of two sectors; industry and agriculture. Our focus being on agriculture, the industrial/service sector is summarized by a constant return to scale production technology that allows production of one item with one unit of labor. The industrial good serves as the numeraire which implies that the wage in these economies is equal to 1. The agricultural sector produces a continuum of goods indexed by  $z \in [0, 1]$  using three factors: land, labor and agrochemicals (pesticides, herbicides, fungicides and the like) directed to control pests and dubbed “pesticides” in the following.<sup>9</sup> Both countries are endowed with total labor force  $L$  and farmland  $N$ , which is the number of individual plots. With  $N$  individuals in farming, industry employs  $L - N$  workers.

Technical coefficients in agriculture differ from one crop to the other, and from one country to the other. More precisely, absent production externality and adverse meteorological or biological events, the mere combination of one unit of labor with one unit of land produces  $\bar{a}(z)$  good  $z$  in Home and  $\bar{a}^*(z)$  in Foreign.<sup>10</sup> The crops are ranked in decreasing order for Home: the relative crop yield  $A(z) \equiv \bar{a}^*(z)/\bar{a}(z)$  satisfies  $A'(z) > 0$ ,  $A(0) < 1$  and  $A(1) > 1$ . Hence, on the basis of these differences in potential yield, Home is more efficient producing goods belonging to  $[0, z_s)$  and Foreign over  $(z_s, 1]$  where  $z_s = A^{-1}(1)$ . However, the production of crops is affected by various factors such as weather conditions (droughts or hail) and potential lethal strains (pests, pathogens), resulting in land productivity lower than its potential. We suppose in the following that a unit plot of crop  $z$  is destroyed with probability  $1 - \psi(z)$  from one (or several) adverse conditions and that, otherwise, with probability  $\psi(z)$  the crop survives and the plot produces the yield  $\bar{a}(z)$  independently of the fate of the other plots in the region.<sup>11</sup> Here,  $\psi(z)$  is a survival probability that depends on many factors, among which a biodiversity factor that depends on the way land is farmed in the country,

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<sup>9</sup>In order to streamline the analysis, we don't consider fertilizers. They are discussed section 5.

<sup>10</sup>Throughout the paper, we index variables belonging to Foreign by superscript “\*”.

<sup>11</sup>Pests and/or meteorological events do not necessarily totally destroy a plot, but rather affect the quantity of biomass produced. This assumption allows for tractability, our random variable being the number of harvested plots rather than the share of harvested biomass.

and more specifically, the intensity of each crop cultivation as measured by the proportion of land devoted to it, i.e.  $B(z) \equiv N(z)/N$  where  $N(z)$  is the number of crop  $z$  plots. The biodiversity effect we consider relates positively the intensity of a crop with the probability that pests affect each individual plot of this crop if farmers do not take appropriate actions to diminish their exposure. This biodiversity effect thus corresponds to a negative production externality between farmers of the same crop. To counteract the impact of external events on her plot, a farmer can rely on a large range of chemicals. All these individual actions put together may in turn have a positive impact on the survival probability of plots by impeding the pest infection in the region, which limits the negative externality due to the intensity of the cultivation. To synthesize these elements in a manageable way, we assume that at the farmer  $i$ 's level, the survival probability of her plot is given by

$$\psi(z, \pi_i, \bar{\pi}(z), B(z)) = \mu_0 \eta(\pi_i, z) / [1 + \hat{\kappa}(\bar{\pi}, z) B(z)] \quad (1)$$

where  $\pi_i$  is the quantity of pesticides used by farmer  $i$ , and  $\bar{\pi}(z)$  the average quantity of pesticides used by the  $N(z) - 1$  other farmers of crop  $z$  in the country. Here,  $\eta(\pi_i, z)$  corresponds to the impact of farmer  $i$ 's pesticides on the resilience of her plot, with  $\eta(0, z) = 1$ ,  $\eta'_z(0, z) > 0$ ,  $\eta''_{zz} < 0$ , and  $\mu_0[1 + \hat{\kappa}(\bar{\pi}, z) B(z)]^{-1}$  the survival probability of her plot if she does not use pesticides, which is nevertheless affected by a cross-external positive impact of the other farmers' treatments, with  $\hat{\kappa}(\bar{\pi}, z) > 0$  and  $\hat{\kappa}'_{\bar{\pi}}(\bar{\pi}, z) < 0$ . Hence, the expected resilience of individual plots decreases with the intensity of the crop  $B$  but increases with the level of pesticides used  $\bar{\pi}$ . In the following, we consider the convenient specifications  $\eta(\pi, z) = e^{\pi(\theta(z) - \pi/2)}$  where  $\theta(z)$  corresponds to the level of pesticides that maximizes the expected yield of the crop, and  $\hat{\kappa}(\bar{\pi}, z) = e^{(\theta(z) - \bar{\pi})^2/2} \kappa(z)$  where  $\kappa(z) \equiv \kappa_0 e^{-\theta(z)^2/2}$  is the lowest cross externality factor for crop  $z$ , when all crop- $z$  farmers use pesticide level  $\theta(z)$ , and  $e^{-(\theta(z) - \bar{\pi})^2/2}$  the negative effect on the crop's resilience induced by an average use of pesticides  $\bar{\pi} \leq \theta(z)$ . We assume that  $\mu_0, \kappa_0$  and  $\theta(z)$  are the same in both countries and such that  $\psi(\cdot) \leq 1$ .<sup>12</sup>

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<sup>12</sup>Parameter  $\mu_0$  may be considered as resulting from factors beyond the control of the collective action of farmers, such as meteorological events, which may differ from one crop to the other and from one country

Because of the externality due to pesticides (on human health and the environment), governments restrict their use. We model the governmental policy as a tax which results in a pesticide's price  $\tau$ . To simplify the following derivations, we suppose that the governments complement this tax policy with a subsidy that allows farmers of crop  $z$  to cope with the environmental regulation: at equilibrium, it is financially neutral for farmers.

We assume that farmers are risk-neutral and may only farm one crop (the one they want) on one unit plot. Given their crop choice, they choose the intensity of the chemical treatment of their field in order to reach expected income

$$r(z) \equiv \max_{\pi} \bar{a}(z)\psi(z, \pi, \bar{\pi}(z), B(z))\bar{p}(z) - \tau\pi + T(z) - w$$

where  $T(z)$  is the subsidy for crop  $z$ , which is considered as a lump sum payment by farmers,  $w$  the other input costs, i.e. the sum of the wage (one unit of labor is necessary to farm a plot of land) and the rental price of land and  $\bar{p}(z) \equiv p(z) + \text{cov}(\tilde{p}(z), \tilde{y}(z))/y(z)$ , the crop  $z$  reference price which differs from the expected market price  $p(z) \equiv E[\tilde{p}(z)]$  to account for the correlation between total production  $\tilde{y}(z)$  and market price  $\tilde{p}(z)$ .<sup>13</sup> The first-order condition leads to an optimal level of pesticides  $\pi(z)$  that satisfies

$$\eta'_{\pi}(\pi(z), z) = \frac{\tau[1 + \hat{\kappa}(\pi(z), z)B(z)]}{\mu_0 \bar{a}(z)\bar{p}(z)}$$

at the symmetric equilibrium between crop  $z$  farmers. As the subsidy allows farmers to break even, we have  $T(z) = \tau\pi(z)$ .<sup>14</sup> Competition in the economy (notably on the land market) implies  $r(z) = 0$  at equilibrium, which gives

$$\pi(z) = \theta(z) - \tau/w \tag{2}$$

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to the other, i.e. we may rather have two functions  $\mu(z)$  and  $\mu^*(z)$  taking different values. However, as competitive advantages are assessed by comparing  $a(z) = \mu_0 \bar{a}(z)e^{\theta(z)}$  to its foreign counterpart, assuming that it is a parameter is innocuous.

<sup>13</sup>As  $\text{cov}(\tilde{p}(z), \tilde{y}(z))/y(z) < 0$ , we have  $\bar{p}(z) < p(z)$ .

<sup>14</sup>For the sake of simplicity, we consider neither the production nor the market of agrochemicals in the following. Implicitly, farmers thus are “endowed” with a large stock of agrochemicals that farming does not exhaust, leading to prices equal to 0.

and implies that the reference price  $\bar{p}(z)$  also corresponds to the farmers' break-even price, which verifies

$$\bar{a}(z)\psi(z)\bar{p}(z) = w \quad (3)$$

where  $\psi(z) \equiv \psi(z, \pi(z), \pi(z), B(z))$ . Defining the tax index  $t \equiv e^{(\tau/w)^2/2}$  and denoting  $a(z) \equiv \bar{a}(z)\mu_0 e^{\theta(z)^2/2}$ , we get

$$\bar{p}(z) = wt[1 + t\kappa(z)B(z)]/a(z) \quad (4)$$

which corresponds to a domestic expected production level

$$y(z) = \frac{a(z)NB(z)}{t[1 + t\kappa(z)B(z)]}. \quad (5)$$

Production of crop  $z$  decreases with  $t$  because of two effects: the corresponding reduction in the use of pesticides has a direct negative impact on the productivity of each plot but also an indirect negative cross-externality effect between plots. Observe that, as environmental taxes are reimbursed at equilibrium, we have, using (3)

$$E[\tilde{p}(z)\tilde{y}(z)] = \bar{p}(z)\bar{a}(z)\psi(z)NB(z) = wNB(z), \quad (6)$$

i.e. that the expected value of the crop  $z$  domestic production is equal to the sum of the wages and the land value involved in its farming.

The representative individuals of the two countries share the same preferences over the goods. Their preferences are given by the Cobb-Douglas utility function

$$U = b \ln x_I + (1 - b) \int_0^1 \alpha(z) \ln \tilde{x}(z) dz - hZ$$

where  $\int_0^1 \alpha(z) dz = 1$  and  $h > 0$ . The first two terms correspond to the utility derived from the consumption of industrial and agricultural goods respectively, while the last term

corresponds to the disutility of the environmental damages caused by a domestic use of

$$Z = N \int_0^1 B(z) \pi(z) dz$$

pesticides by farmers. The corresponding demand for the industrial good is

$$x_I = bR$$

where  $R$  is the revenue per capita. The rest of the revenue,  $(1 - b)R$ , is spent on agricultural goods with demand for crop  $z$  given by

$$\tilde{x}(z) = \alpha(z)(1 - b)R/\tilde{p}(z)$$

where  $\tilde{p}(z)$  depends on the realized production level  $\tilde{y}(z)$  and  $\alpha(z)$  is the share of the food spending devoted to crop  $z$ . We assume that consumers are risk-neutral and thus evaluate their *ex ante* welfare at the average consumption level of crop  $z$ ,  $x(z) = E[\tilde{x}(z)]$ , which also corresponds to reference price  $\bar{p}(z)$ .<sup>15</sup> Their indirect utility function is given by<sup>16</sup>

$$V(R, Z) = \ln(R) - (1 - b) \int_0^1 \alpha(z) \ln \bar{p}(z) dz - hZ. \quad (7)$$

where  $R = [L - N + wN]/L$  since there is no profit at equilibrium.

The government determines the optimal policy by maximizing this utility, taking account of the relationship between pesticides and the value of land.

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<sup>15</sup>This is implied by the fact that at equilibrium total spending is equal to the total value of crop  $z$  which in expectation is equal to  $\bar{p}(z)y(z)$ . Observe that as demand functions are non-linear,  $E[\tilde{x}(z)] = E[1/\tilde{p}(z)]\alpha(z)(1 - b)R$  and thus  $\bar{p}(z) = E[1/\tilde{p}(z)]^{-1}$

<sup>16</sup>Up to a constant given by  $b \ln(b) + (1 - b) \int_0^1 \alpha(z) \ln \alpha(z) dz$ .

## 2.1 Equilibrium in autarky

The rental price of the agricultural land in autarky,  $w_A - 1$ , is derived from the equilibrium condition on the market for industrial goods. Indeed, we must have

$$bLR = b(Nw_A + L - N) = L - N$$

which gives

$$w_A = (\ell - 1)(1 - b)/b$$

where  $\ell \equiv L/N > 1$ . Consequently, the value of land is positive if  $N < (1 - b)L$ , a condition assumed to hold in the following. Observe that this value depends neither on the use of pesticides nor on the crops' prices and is the same in both countries in spite of their differences in terms of crop yields. This is due to the Cobb-Douglas preferences (in addition to the constant productivity in the industrial sector). Total domestic revenue is given by

$$LR = Nw_A + L - N = N(\ell - 1)/b$$

for both countries. Market equilibrium on the crop  $z$  market implies that total expenses are equal to total production cost, i.e.

$$\alpha(z)(1 - b)LR = NB(z)w_A$$

which gives  $B(z) = \alpha(z)$ . The average production for Home is thus

$$y_A(z) = \frac{a(z)N\alpha(z)}{t[1 + t\kappa(z)\alpha(z)]}, \quad (8)$$

and for Foreign  $y_A^*(z) = A(z)y_A(z)$ , and the corresponding break-even prices

$$\bar{p}_A(z) = \frac{(1 - b)(\ell - 1)t[1 + t\kappa(z)\alpha(z)]}{ba(z)} \quad (9)$$

and  $\bar{p}_A^*(z) = \bar{p}_A(z)/A(z)$ . Using (2) and  $\tau/w = \sqrt{2 \ln t}$ , we get

$$Z = N \int_0^1 \alpha(z) \theta(z) dz - N \sqrt{2 \ln t}.$$

Finally, the optimal level of pesticide (or, equivalently, the optimal pesticides tax) is determined by maximizing the utility of the representative consumer (7) which reduces to

$$\min_t (1 - b) \int_0^1 \alpha(z) \ln(t[1 + t\kappa(z)\alpha(z)]) dz + hZ,$$

a program that applies to both countries. We obtain that the optimal tax index in autarky,  $t_A$ , solves

$$\sqrt{2 \ln t_A} \left[ 1 + \int_0^1 \frac{t_A \kappa(z) \alpha(z)^2}{1 + t_A \kappa(z) \alpha(z)} dz \right] = \frac{Nh}{1 - b}. \quad (10)$$

As  $\kappa(z) = \kappa_0 e^{-\theta(z)^2/2}$ , the optimal tax decreases when  $\kappa_0$  increases, with a maximum given by  $\tau_A = (L - N)h/b$ .

While acreage and pesticide levels are the same in both countries, the average production of each is different because of the differences in crop yields. The revenue being the same in both countries, crop demands are identical but because average production levels are different, break-even prices are also different.

## 2.2 Free trade equilibrium

First consider the equilibrium on the market for the industrial good. We have

$$b(Nw + L - N + N^*w^* + L^* - N^*) = L - N + L^* - N^*$$

where  $L = L^*$  and  $N = N^*$ . We obtain

$$w + w^* = 2(\ell - 1)(1 - b)/b$$



hence, the worldwide agricultural revenue is the same as in autarky. This is also the case for the total revenue, given by

$$LR + L^*R^* = N[w + w^* + 2(\ell - 1)] = 2N(\ell - 1)/b.$$

As in autarky, the share of the agricultural sector of this revenue is unchanged, given by  $1 - b$ . For Home, it results in a per-individual revenue given by

$$R = (\ell - 1)[1 + 2q(1 - b)/b], \quad (11)$$

which depends on the fraction  $q = w/(w + w^*)$  obtained at equilibrium. This share depends on crop yields and on the environmental tax policies implemented in each country.

In addition to tax policies, competitive advantages also depend on biodiversity effects that evolve with the way land is farmed. Indeed, for a given tax level, because of the production externality effect, the higher the intensity of the farming of a crop, the lower the average productivity of the land: intensification undermines the competitive advantages apparent under autarky. More precisely, if crop  $z$  is produced by Home only, the market equilibrium condition implies that worldwide expenses on crop  $z$  are equal to total production cost, i.e.,

$$2\alpha(z)(\ell - 1)(1 - b)N/b = B(z)wN$$

which can be written as  $\alpha(z)/q = B(z)$ . Opening to trade could thus correspond to a large increase of the acreage devoted to that crop: e.g., if  $q = 1/2$ , the total farmland doubles which may seriously impair Home's land productivity for crop  $z$ . On the other hand, crop  $z$  is produced by both countries if their break-even prices are equal, or using (4), if

$$A(z) = \frac{w^*}{w} \frac{t^*}{t} \frac{1 + t^*\kappa(z)B^*(z)}{1 + t\kappa(z)B(z)}. \quad (12)$$

At market equilibrium, worldwide expenditure on crop  $z$  is equal to the sum of production

costs of the two countries:

$$2\alpha(z)N(\ell - 1)(1 - b)/b = wNB(z) + w^*N^*B^*(z),$$

which can also be written as

$$\alpha(z) = qB(z) + q^*B^*(z). \quad (13)$$

Crop  $z$  is thus produced by both countries if there exist  $B(z) > 0$  and  $B^*(z) > 0$  which solve (12) and (13). As stated formally in the following proposition, this is true for a whole range of crops. More precisely,

**Proposition 1** *Specialization is incomplete under free trade: Assuming  $\kappa_0$  is not too large, both countries produce crops belonging to  $(\underline{z}, \bar{z})$ ,  $0 \leq \underline{z} < \bar{z} \leq 1$  satisfying*

$$A(\bar{z}) = \frac{t^*}{t} \frac{q^* + t^*\kappa(\bar{z})\alpha(\bar{z})}{q} \quad (14)$$

and

$$A(\underline{z}) = \frac{t^*}{t} \frac{q^*}{q + t\kappa(\underline{z})\alpha(\underline{z})}. \quad (15)$$

The intensity of these crops is given by

$$B(z) = \chi(z)[1 - q\phi(z)]/q \quad (16)$$

where

$$\phi(z) \equiv \frac{1 + A(z)t/t^*}{1 + \alpha(z)t^*\kappa(z)}, \quad (17)$$

$$\chi(z) \equiv \frac{1 + t^*\alpha(z)\kappa(z)}{t\kappa(z)[A(z)t/t^* + t^*/t]} \quad (18)$$

for Home and symmetric expressions hold for Foreign (with  $A(z)$  replaced by  $1/A(z)$ ). Crops belonging to  $[0, \underline{z}]$  ( $[\bar{z}, 1]$ ) are produced by Home (Foreign) only, with intensity  $B(z) = \alpha(z)/q$ .

**Proof:** see the appendix.

Using (14) and (15) with  $\kappa_0 = 0$ , we end up with  $A(\bar{z}) = A(\underline{z}) = (q^*/q)(t^*/t)$  and thus

a unique threshold index and complete specialization. With  $\kappa_0 > 0$ , we have  $A(\underline{z}) < A(\bar{z})$  and since  $A$  is strictly increasing,  $\underline{z} < \bar{z}$ : albeit technical differences exist between the two countries, comparative advantages are trimmed by the negative externality that affects national production of each country.

Using (5) and (16), we obtain that Home's expected production of crop  $z \in (\underline{z}, \bar{z})$  is

$$y_T(z) = Na(z) \frac{q^* + \alpha(z)\kappa(z)t^* - qA(z)t/t^*}{t\kappa(z)[qt^* + q^*t + tt^*\alpha(z)\kappa(z)]}.$$

Using the symmetric expression for Foreign, we obtain that total expected production under free trade is given by

$$\begin{aligned} \bar{y}_T^W &= N \frac{\alpha(z)[a^*(z)t_T/t_T^* + a(z)t_T^*/t_T]}{qt_T^* + q^*t_T + \alpha(z)t_T t_T^* \kappa(z)} \\ &= y_A(z) \frac{t_A}{t_T} \frac{1 + \alpha(z)t_A \kappa(z)}{1 + \alpha(z)t_T^* \kappa(z) - q(1 - t_T^*/t_T)} + y_A^*(z) \frac{t_A}{t_T^*} \frac{1 + \alpha(z)t_A \kappa(z)}{1 + \alpha(z)t_T \kappa(z) - q^*(1 - t_T/t_T^*)}. \end{aligned} \quad (19)$$

Observe that with unchanged tax ratios ( $t_T = t_A$ ) in both countries, we would have  $\bar{y}_T^W = y_A^W$ . More generally, compared to the production of these crops under autarky, trade operates as if two distortive effects were at work on domestic productions: a direct tax effect and a cross-externality effect. The first one, given by the ratio  $t_A/t_T$ , measures the impact of the change in the environmental policy on the use of pesticides. The second one also reflects the impact of the policy change on the production externality, but it is not a simple ratio: there is a correcting term given by  $q(1 - t_T^*/t_T)$  which also depends on the relative wealth.

For these crops produced by both countries, the break-even price is

$$\bar{p}_m(z) = \frac{2(1-b)(\ell-1)}{ba(z)} \frac{q(t^* - t) + t[1 + \alpha(z)t^* \kappa(z)]}{t^*/t + A(z)t/t^*}. \quad (20)$$

For the other crops, the corresponding expected productions and break-even prices are given by

$$y(z) = \frac{a(z)N\alpha(z)}{t[q + t\kappa(z)\alpha(z)]}$$

and

$$\bar{p}_s(z) = \frac{2(\ell - 1)(1 - b)t[q + t\kappa(z)\alpha(z)]}{ba(z)} \quad (21)$$

for all  $z \leq \underline{z}$  and by symmetric expressions for all crops  $z \geq \bar{z}$ .

From these preliminary results, it is possible to isolate a first effect due to the production externality by supposing that the environmental taxes and the agricultural revenues of the two countries are unchanged at equilibrium.

**Lemma 1** *Suppose that  $t_T = t_T^* = t_A$  and  $q = 1/2$  at equilibrium under free trade. Then, the break-even prices of the crops produced by only one country are higher for this country than under autarky and are reduced for the other country. For crops produced by both countries, the price for Home (Foreign) is lower (larger) for all crops  $z \in (A^{-1}(1), \bar{z}]$  and larger (lower) for all  $z \in [\underline{z}, A^{-1}(1))$ .*

**Proof:** see the appendix.

These increases in break-even prices affecting crops produced more intensively under trade than under autarky are the consequences of the cross-externality effects between plots: while the input prices do not change by assumption, the marginal production cost increases, whereas in DFS it is constant.

Of course, environmental taxes and the sharing of the total agricultural revenue evolve with international trade. Using (2), we obtain that the level of pesticides under free trade is given by

$$Z_T = \frac{N}{q} \left\{ \int_0^{\underline{z}} \alpha(z)\theta(z)dz + \int_{\underline{z}}^{\bar{z}} \chi(z)[1 - q\phi(z)]\theta(z)dz \right\} - N\sqrt{2 \ln t} \quad (22)$$

which depends on the crops farmed at equilibrium and on how the worldwide agricultural revenue is shared.

To determine this share, we can use the fact that the domestic revenues come from the sale of the goods produced nationally.<sup>17</sup> On interval  $[0, \underline{z}]$ , all revenues spent are collected

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<sup>17</sup>The same expression can be derived using the equilibrium condition on the land market.

by Home, while it is only a share  $s(z)$  of them on  $[\underline{z}, \bar{z}]$ . We thus have

$$LR = L - N + (1 - b)(LR + L^*R^*) \left\{ \int_0^{\underline{z}} \alpha(z) dz + \int_{\underline{z}}^{\bar{z}} s(z) \alpha(z) dz \right\}$$

which simplifies to

$$q = \int_0^{\underline{z}} \alpha(z) dz + \int_{\underline{z}}^{\bar{z}} s(z) \alpha(z) dz \quad (23)$$

where  $s(z) \equiv y_T(z)/y_T^W(z)$  corresponds to Home's share of expected production relative to the total. Using (6), (13) and (16), we get

$$s(z) = \frac{qB(z)}{qB(z) + q^*B^*(z)} = \frac{qB(z)}{\alpha(z)} = \frac{[1 - q\phi(z)]\chi(z)}{\alpha(z)} \quad (24)$$

and thus

$$q = \frac{\int_0^{\underline{z}} \alpha(z) dz + \int_{\underline{z}}^{\bar{z}} \chi(z) dz}{1 + \int_{\underline{z}}^{\bar{z}} \phi(z) \chi(z) dz}. \quad (25)$$

### 3 Environmental tax policy and trade

The free trade equilibrium depends on the environmental tax policies  $t$  and  $t^*$  that determine the sharing of the worldwide agricultural revenue between the countries. We model this problem as a Nash equilibrium of a two-stage game where Home and Foreign governments choose simultaneously their tax policies in the first stage, knowing that in the second stage farmers decide which crops to sow and how much pesticide to use. Home's government problem when defining its tax policy corresponds to the following program:

$$\max_t \ln(R) - (1 - b) \left\{ \int_0^{\underline{z}} \alpha(z) \ln \bar{p}_s(z) dz + \int_{\underline{z}}^{\bar{z}} \alpha(z) \ln \bar{p}_m(z) dz + \int_{\bar{z}}^1 \alpha(z) \ln \bar{p}_s^*(z) dz \right\} - hZ \quad (26)$$

where  $\bar{p}_s^*(z)$  is given by (21) with  $t$  and  $q$  replaced by  $t^*$  and  $q^* = 1 - q$ . The optimal environmental policy resulting from this program depends on  $t^*$ : maximizing (26) gives Home's best-response to Foreign's policy  $t^*$ . We suppose that Foreign's government is the symmetric situation, i.e. that opening the borders to trade results in a Nash equilibrium

of a game where both governments act simultaneously in a strategic way. For the sake of argument, we consider two cases in the following: one in which governments ignore the relationship between their tax policies and their share of the worldwide agricultural revenue, called “non-strategic” free trade equilibrium, and the more realistic case where they reckon that  $q$  and thus  $R$  depend on  $t$  and  $t^*$ , which leads to trade equilibrium dubbed “strategic”. One may easily show that  $q$  is negatively related to  $t$  and positively related to  $t^*$ . Indeed, a total differentiation of (23) yields, using  $s(\bar{z}) = 0$  and  $s(\underline{z}) = 1$ ,

$$\begin{aligned}\frac{dq}{dt} &= \int_{\underline{z}}^{\bar{z}} \frac{ds(z)}{dt} \alpha(z) dz = \int_{\underline{z}}^{\bar{z}} \frac{d}{dt} [\chi(z) - q\phi(z)\chi(z)] dz \\ &= q \frac{\int_{\underline{z}}^{\bar{z}} [B(z)(d\chi(z)/dt)/\chi(z) - \chi(z)(d\phi(z)/dt)] dz}{1 + \int_{\underline{z}}^{\bar{z}} \phi(z)\chi(z) dz}\end{aligned}\quad (27)$$

where it is straightforward from (18) and (17) that  $d\chi(z)/dt < 0$  and  $d\phi(z)/dt > 0$ . Hence, we have  $dq/dt < 0$  and since  $q + q^* = 1 - b$ ,  $dq^*/dt = -dq/dt > 0$ . Because of the strategic substitutability of these taxes, pesticides are used more intensively at the strategic trade equilibrium than when governments act non strategically. To detail this competition effect and assess its interaction with the biodiversity externality that affects production, we consider in the following the case where  $\alpha(z) = 1$  and  $\theta(z) = \theta$  for all  $z$  which implies that  $\kappa(z) = \kappa_0 e^{-\theta^2/2} \equiv \kappa$  for all  $z$ . Hence, neither the demand nor the externality on consumers’ utility distinguishes crops, so we have

$$Z = N\theta - N\sqrt{2\ln t}$$

whatever the case at hand. We analyze the two types of free trade equilibria (non-strategic and strategic) assuming first that there is no biodiversity effect, i.e.  $\kappa = 0$ . We then introduce the negative production externality ( $\kappa > 0$ ).

### 3.1 Trade without biodiversity effect

When  $\kappa = 0$ , we obtain using (10),  $\sqrt{2 \ln t_A} = \tau_A/w_A$  and  $w_A = (\ell - 1)(1 - b)/b$ , that the environmental tax under autarky is given by

$$\tau_A = (L - N)h/b.$$

Under free trade, each country specializes on one segment of the range of crops delimited by threshold  $z_s$ , which satisfies

$$A(z_s) = \frac{q^* t^*}{qt}$$

with

$$\int_0^{z_s} B(z) dz = \int_0^{z_s} (1/q) dz = 1$$

hence  $q = z_s$ : Home's share of the worldwide agricultural revenue is equal to the range of crops produced domestically. We obtain that  $z_s$  solves  $\xi(z_s) = t^*/t$  where

$$\xi(z) \equiv A(z)z/(1 - z)$$

is strictly increasing. In the non-strategic situation, governments do not take into account the effect of their environmental taxes on the sharing of the agricultural revenue. The effect of the tax policies on  $z_s, q$  and  $R$  are neglected when solving (26). The problem simplifies to

$$\min_t (1 - b)q \ln t - hN\sqrt{2 \ln t}$$

where  $q$  is considered as a constant. The first-order condition leads to an optimal tax that solves

$$\sqrt{2 \ln t} = \frac{Nh}{q(1 - b)}. \quad (28)$$

Using  $q = z_s$ , we get  $t = e^{(\frac{Nh}{(1-b)z_s})^2/2}$  and thus a threshold crop that solves  $\xi(z_s) =$

$e^{\frac{(Nh)^2(1-2z_s)}{2[(1-b)z_s(1-z_s)]^2}}$ . As  $\sqrt{2 \ln t} = \tau/w$  and  $w = 2qw_A$ , (28) simplifies to

$$\frac{\tau}{w} = \frac{\tau}{2qw_A} = \frac{\tau_A}{qw_A}$$

hence  $\tau = 2\tau_A$  whatever the country's share of the worldwide agricultural revenue. Stated formally:

**Proposition 2** *Suppose that there are no biodiversity effects. Then, at the non-strategic trade equilibrium, the environmental tax is doubled compared to under autarky.*

The intuition is as follows. The environmental policy affects only crops produced domestically. As the range of domestic products is smaller under free trade than under autarky, the impact of the environmental policy on consumer welfare is reduced on the consumption side (prices affected by the tax are only those produced by Home) while it is unchanged on the environmental side. It is thus optimal to raise the tax compared to under autarky. In the case of the specific parameters at hand, the autarky program simplifies to  $\min_t (1-b) \ln t - hN\sqrt{2 \ln t}$  and thus the benefits of pesticides are  $1/q$  as much larger than under free trade. The resulting tax level  $\tau_A/w_A$  is  $q$  times the one under free trade,  $\tau/w$ , but as the rental price of land has changed with free trade, leading to a production cost equal to  $q2w_A$ , the nominal tax rate is doubled.

Observe that the resulting situation is not Pareto optimal: indeed, if the two countries could agree on tax levels, each would have to account for the price effect of its tax on the other country's consumers. In our setup, the resulting Pareto optimal tax level is the autarky one.<sup>18</sup>

Now suppose that governments are strategic in the sense that they take into account the effect of the tax on their shares of agricultural revenue. Using (11) and (26) we obtain that Home's best-response to  $t^*$  solves

$$\max_{t,q} \left\{ \ln \left( 1 + \frac{2q(1-b)}{b} \right) - (1-b) \left[ \int_0^q \ln \bar{p}_s(z) dz + \int_q^1 \ln \bar{p}_s^*(z) dz \right] - hZ : q = \xi^{-1} \left( \frac{t^*}{t} \right) \right\}.$$

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<sup>18</sup>Indeed, for any sharing  $(q, 1-q)$  of the agricultural revenue  $2w_A$ , the Pareto optimal tax levels solve  $\min_t 2(1-b)q \ln t - hN\sqrt{2 \ln t}$  and the equivalent program for Foreign.



It is implicitly defined by

$$\sqrt{2 \ln t} \left\{ q + \left[ \frac{2}{b + 2q(1-b)} \right] \frac{t^*}{t \xi'(z_s)} \right\} = \frac{Nh}{1-b}. \quad (29)$$

At a symmetric equilibrium, i.e.  $q = 1/2 = z_s$ ,  $t = t^*$ , which implies that  $A(1/2) = 1$  and thus  $\xi'(z_s) = A'(1/2) + 4$ , we get

$$\sqrt{2 \ln t} = \frac{Nh}{1-b} \left[ 1 + \frac{A'(1/2)}{A'(1/2) + 8} \right].$$

The following proposition characterizes the optimal policy at equilibrium:

**Proposition 3** *Suppose that there are no biodiversity effects and that  $A(1/2) = 1$ . Then, at the symmetric strategic trade equilibrium, the environmental tax  $\tau$  verifies  $2\tau_A > \tau \geq \tau_A$ , with  $\tau = \tau_A$  in the limit case where  $A'(1/2) = 0$ . Moreover, the steeper the comparative advantage function  $A(z)$ , the larger the environmental tax.*

**Proof:** see the appendix.

When comparative advantages are not too different, allowing farmers to use more pesticides could have a large impact on the country's market share of agricultural products. At the symmetric equilibrium, countries do not gain market share, but contrary to the usual strategic trade setup, this unproductive competition in terms of market share results in a situation which is a Pareto improvement compared to the non-strategic one.

### 3.2 Biodiversity effects

Biodiversity effects create two countervailing distortions in the governments' trade-off we have described above: On the one hand, as specialization increases the production externality, governments should be induced to lower the tax on pesticides. On the other hand, as the externality limits specialization, the effect of the tax on prices concerns a reduced set of crops, which should induce governments to increase the tax. To give a comprehensive appraisal of these countervailing effects, we focus on the symmetric case and refine further

our setup by considering that the relative potential yield function is given by

$$A(z) = \frac{1 + m(2z - 1)}{1 - m(2z - 1)} \quad (30)$$

where  $0 < m < 1$ . We have  $0 < A(0) < 1$ ,  $A(1) > 1$ ,  $A(1/2) = 1$  and

$$A'(z) = \frac{4m}{[1 - m(2z - 1)]^2}.$$

Hence, the larger  $m$ , the larger the discrepancy between the countries' relative potential yield away from  $z = 1/2$  (graphically, the relative potential yield curve becomes steeper when  $m$  increases).

At a symmetric equilibrium, threshold crops given by (14) and (15) simplify to

$$\bar{z} = \frac{1}{2} + \frac{t\kappa}{2m(1 + t\kappa)} \quad (31)$$

and

$$\underline{z} = \frac{1}{2} - \frac{t\kappa}{2m(1 + t\kappa)}. \quad (32)$$

They are equally distant from the center of the range of crops ( $1/2$ ), and the length of the subset of crops produced by both countries,

$$\bar{z} - \underline{z} = \frac{1}{m} \frac{t\kappa}{1 + t\kappa}, \quad (33)$$

increases with  $\kappa$  and  $t$  and decreases with the relative potential yield parameter  $m$ .

In the non-strategic case, the condition that determines  $t$  at the symmetric equilibrium can be written as  $(\partial V / \partial t)_{t^*=t} = 0$  where

$$\left. \frac{\partial V}{\partial t} \right|_{t^*=t} = -\frac{1-b}{t} \left[ \frac{1+4t\kappa}{1+2t\kappa} \underline{z} + (\bar{z} - \underline{z}) \frac{1+2t\kappa}{2(1+t\kappa)} \right] - h \frac{dZ}{dt}. \quad (34)$$

The last term corresponds to the environmental impact on consumers, which is positive since  $dZ/dt < 0$ . It leads the government to increase the environmental tax. The bracketed

term is composed of two elements, the first one corresponding to the price effect on the goods produced locally and the second one to the price effect on the goods produced by both countries. In these terms, biodiversity effects are ambiguous. Indeed, using (32) and (33), the effect on goods produced locally can be rewritten as

$$\frac{1 + 4t\kappa}{1 + 2t\kappa} \underline{z} = \left(1 + \frac{2t\kappa}{1 + 2t\kappa}\right) \left(\frac{1}{2} - \frac{\bar{z} - \underline{z}}{2}\right)$$

which shows that compared to the case where  $\kappa = 0$ , intensification of farming has an ambiguous impact: in the first bracket, the fraction  $2t\kappa/(1 + 2t\kappa)$  tends to reduce the tax on the crops produced locally while the second bracketed term highlights that the range of crops specific to Home is not half of the total but is reduced by  $(\bar{z} - \underline{z})/2$ , which tends to increase the tax. This effect due to the decrease in the range of specific crops exceeds the one concerning crops produced by both countries: indeed, we have

$$\frac{\bar{z} - \underline{z}}{2} \left(1 + \frac{2t\kappa}{1 + 2t\kappa} - \frac{1 + 2t\kappa}{1 + t\kappa}\right) = \frac{\bar{z} - \underline{z}}{2} \left(\frac{2t\kappa}{1 + 2t\kappa} - \frac{t\kappa}{1 + t\kappa}\right) > 0.$$

The fact that both countries are producing crops belonging to  $(\underline{z}, \bar{z})$  thus tends to increase the tax level compared to the case where  $\kappa = 0$ . It counteracts the intensification effects on specific crops. As a result, the environmental tax could be larger or lower than  $2\tau_A$ , depending on the relative potential yields of crops. More precisely, we have the following result:

**Proposition 4** *Suppose that the relative potential yield function is given by (30). Then, at the symmetric non-strategic trade equilibrium, biodiversity effects result in a reduction of the environmental tax compared to the case where  $\kappa = 0$  unless  $m$  is very small. Overall, the environmental tax is greater than under autarky.*

**Proof:** see the appendix.

When the discrepancy in relative potential yields is large between the two countries, specialization is important (the range of crops produced by both is relatively small), and the cross externality effect is optimally contained by an intensive use of pesticides.

Finally, in the strategic case, there is a marginal effect of the environmental tax on the share of the agricultural revenue that induces governments to reduce their environmental tax. Indeed, the marginal effect of the tax policy on welfare is given by

$$\frac{dV}{dt} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial q} \frac{dq}{dt}$$

which entails an additional term compared to the non-strategic case, where

$$\frac{\partial V}{\partial q} = \frac{2(1-b)}{b+2q(1-b)} - (1-b) \left[ \frac{\underline{z}}{q+t\kappa} - \frac{1-\bar{z}}{1-q+t^*\kappa} + \frac{(\bar{z}-\underline{z})(t^*-t)}{q(t^*-t)+t(1+t^*\kappa)} \right]. \quad (35)$$

The first term corresponds to the direct effect on welfare due to the increase in revenue while the remaining terms concern the effects on the price of crops produced domestically, abroad and by both countries respectively. At a symmetric equilibrium, the price effects cancel out, leading to  $(\partial V/\partial q)_{t^*=t, q=1/2} = 2(1-b)$ . Using (30), we obtain that (27) is expressed as

$$\left. \frac{dq}{dt} \right|_{t^*=t, q=1/2} = - \frac{3+9t\kappa+4(t\kappa)^2}{12t(1+t\kappa)[m(1+t\kappa)+1]} \quad (36)$$

which decreases with  $\kappa$ : the greater the biodiversity effects, the stronger the negative impact of the environmental tax on the share of the agricultural revenue. However, we show in the appendix that

**Proposition 5** *Suppose that the relative potential yield function is given by (30). Then, at the symmetric strategic trade equilibrium the environmental tax is generally greater than under autarky.*

**Proof:** see the appendix.

## 4 Price volatility

To investigate the biodiversity effects on market price volatility, we can derive some characteristics of the price distributions using the break-even prices, the demand functions and the probability distributions of the crops' production.

With a survival probability  $\psi(z)$  and a per plot yield  $\bar{a}(z)$ , the crop  $z$  production of a country  $\tilde{y}(z)$  follows a binomial distribution with mean  $y(z) = \bar{a}(z)NB(z)\psi(z)$  and variance  $V(\tilde{y}(z)) = \bar{a}(z)^2NB(z)\psi(z)(1 - \psi(z))$ . More precisely, denoting  $\mu(z) = \mu_0 e^{\theta(z)^2/2}$ , we have

$$\psi(z) = \frac{\mu(z)}{t[1 + t\kappa(z)B(z)]}$$

which yields

$$V(\tilde{y}(z)) = y(z)^2 \frac{t[1 + t\kappa(z)B(z)] - \mu(z)}{\mu(z)NB(z)}. \quad (37)$$

Denoting  $v(\tilde{X}) \equiv \sigma(\tilde{X})/E[\tilde{X}]$  the coefficient of variation of random variable  $\tilde{X}$ , i.e. the ratio of its standard deviation to its expected value, we get

$$v(\tilde{y}(z)) = \left[ \frac{t[1 + t\kappa(z)B(z)] - \mu(z)}{\mu(z)NB(z)} \right]^{1/2}. \quad (38)$$

For crops farmed in both countries, as  $V(\tilde{y}^W(z)) = V(\tilde{y}_T(z)) + V(\tilde{y}_T^*(z))$ , we obtain using (37)

$$v(\tilde{y}^W(z))^2 = s(z)^2 v(\tilde{y}(z))^2 + s^*(z)^2 v(\tilde{y}^*(z))^2$$

which gives, using (24)

$$v(\tilde{y}^W(z)) = \left[ \frac{q^2 B(z)(t - \mu(z) + t^2 \kappa(z)B(z)) + q^{*2} B^*(z)(t^* - \mu(z) + t^{*2} \kappa(z)B^*(z))}{\mu(z)N\alpha(z)^2} \right]^{1/2}. \quad (39)$$

Observe also that since the probability of the Bernoulli distribution  $\psi(z)$  doesn't depend on  $N$ , the binomial distribution that characterizes crop  $z$  production converges to the normal distribution  $\mathcal{N}(y(z), \sigma(\tilde{y}(z)))$  when  $N$  is large.

The break-even price differs from the average market price because of the correlation between prices and quantities, i.e.  $\bar{p}(z) = p(z) + \text{cov}(\tilde{p}(z), \tilde{y}(z))/y(z) < p(z)$ . It corresponds to the price for the average quantity  $y(z)$  and, given consumer demand, to the harmonic mean of prices:  $\bar{p}(z) = E[1/\tilde{p}(z)]^{-1}$ . As a consequence  $\bar{p}(z)$  corresponds to crop  $z$ 's median price: indeed,  $\Pr\{\tilde{p}(z) \leq \bar{p}(z)\} = \Pr\{\tilde{y}(z) \geq y(z)\} = 1/2$  since the normal distribution is symmetric (the expected quantity is also the median one). As the median crop price is lower

than the expected crop price, the price distributions of crops are asymmetric.

This asymmetry is also revealed by the upper and lower bounds of confidence intervals implied by the distributions of crop production. Denoting by  $y_d^\gamma(z)$  and  $y_u^\gamma(z)$  the lower and upper quantity bounds of the confidence interval at confidence level  $1 - \gamma$ , the corresponding price bounds are derived from the demand function: we have  $p_u^\gamma(z) \equiv \alpha(z)(1 - b)LR/y_d^\gamma(z)$  and  $p_d^\gamma(z) \equiv \alpha(z)(1 - b)LR/y_u^\gamma(z)$  since  $\Pr\{y_d^\gamma(z) \leq \tilde{y}(z) \leq y_u^\gamma(z)\} = \Pr\{p_d^\gamma(z) \leq \tilde{p}(z) \leq p_u^\gamma(z)\}$ . Because of the symmetry of the quantity distributions,  $y_d^\gamma(z)$  and  $y_u^\gamma(z)$  are equally distant from  $y(z)$ , but since prices and quantities are inversely related, this is not the case for  $p_d^\gamma(z)$  and  $p_u^\gamma(z)$ . The following proposition completes these general features of the price distributions with some useful approximations:<sup>19</sup>

**Proposition 6** *The expected value and the standard deviation of crop prices are approximated by*

$$p(z) \approx \bar{p}(z)[1 + v(\tilde{y}(z))^2] \quad (40)$$

and

$$\sigma(\tilde{p}(z)) \approx \bar{p}(z)v(\tilde{y}(z))(1 - v(\tilde{y}(z))^2)^{1/2}.$$

Confidence intervals at confidence level  $1 - \gamma$  are delimited by  $p_u^\gamma(z) = p(z) + s_u^\gamma\sigma(\tilde{p}(z))$  and  $p_d^\gamma(z) = p(z) + s_d^\gamma\sigma(\tilde{p}(z))$  with

$$s_u^\gamma \approx \frac{v(\tilde{y}(z)) + s_\gamma}{(1 - v(\tilde{y}(z))^2)^{1/2}(1 - s_\gamma v(\tilde{y}(z)))} \quad (41)$$

and

$$s_d^\gamma \approx \frac{v(\tilde{y}(z)) - s_\gamma}{(1 - v(\tilde{y}(z))^2)^{1/2}(1 + s_\gamma v(\tilde{y}(z)))} \quad (42)$$

where  $s_\gamma \equiv \Phi^{-1}(1 - \gamma/2)$ . They are approximately equal to

$$p_u^\gamma(z) \approx \bar{p}(z) \left( 1 + v(\tilde{y}(z))^2 + v(\tilde{y}(z)) \frac{v(\tilde{y}(z)) + s_\gamma}{1 - s_\gamma v(\tilde{y}(z))} \right) \quad (43)$$

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<sup>19</sup>Where  $\Phi$  denotes the cumulative distribution function of the standard normal distribution.

and

$$p_d^\gamma(z) \approx \bar{p}(z) \left( 1 + v(\tilde{y}(z))^2 + v(\tilde{y}(z)) \frac{v(\tilde{y}(z)) - s_\gamma}{1 + s_\gamma v(\tilde{y}(z))} \right). \quad (44)$$

**Proof:** see the appendix.

Because prices and quantities are inversely related, we have  $s_d < s_u$ , i.e the price distribution is skewed to the right: its right tail is longer and fatter than its left tail. The consequences on volatility are that the chances that a crop price is very low compared to the expected price, i.e.,  $\tilde{p}(z) \leq \bar{p}(z) < p(z)$ , are larger than the chances of a high price, i.e.  $\tilde{p}(z) > p(z)$ , since  $1/2 = \Pr\{\tilde{p}(z) \geq \bar{p}(z)\} > \Pr\{\tilde{p}(z) > p(z)\}$ , but the possible range of high prices is wider than the range of low prices:  $p_u(z) - p(z) > p(z) - p_d(z) > \bar{p}(z) - p_d(z)$ .<sup>20</sup>

Proposition 6 also describes how the impact of trade on the general features of the price distribution is captured through the changes in the median price and the coefficient of variation of the crops' distributions given by (38) and (39). At a symmetric equilibrium with  $\alpha(z) = 1, \theta(z) = \theta$  and  $q = 1/2$ , which implies  $\kappa(z) = \kappa$  and  $\mu(z) = \mu$  for all  $z$ , we get, using (16), (17) and (18), that

$$B(z) = \frac{1 + 2t\kappa - A(z)}{t\kappa[1 + A(z)]} \quad (45)$$

which allows us to obtain that (39) simplifies to

$$v(\tilde{y}(z)) = \left[ \frac{(1 + t\kappa)(1 + 2t\kappa) - \mu\kappa}{2\mu N\kappa} - \frac{2(1 + t\kappa)^2}{\mu N\kappa} \frac{A(z)}{[1 + A(z)]^2} \right]^{1/2}.$$

As  $A(z)/[1 + A(z)]^2$  is bell-shaped with a maximum at  $A(z) = 1$ , the volatility of crops produced by both countries is thus decreasing over the range  $[\underline{z}, 1/2)$  and increasing over  $(1/2, \bar{z}]$ , the minimum being

$$v(\tilde{y}(1/2)) = \left( \frac{t(1 + t\kappa) - \mu}{2\mu N} \right)^{1/2}.$$

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<sup>20</sup>This is of course is dependent on the convexity of the demand function. Indeed, the condition  $p_u(z) - p(z) > p(z) - p_d(z)$ , is equivalently written  $p(z) < [p_u(z) + p_d(z)]/2 = [D^{-1}(y_d(z)) + D^{-1}(y_u(z))]/2$ . As  $p(z) > \bar{p}(z) = D^{-1}(y(z))$  where  $y(z) = [y_d(z) + y_u(z)]/2$ , a necessary condition is  $D^{-1}([y_d(z) + y_u(z)]/2) \leq [D^{-1}(y_d(z)) + D^{-1}(y_u(z))]/2$ , hence  $D^{-1}(y)$  must be convex from the Jensen's inequality.

This is due to a diversification effect that is maximum for  $z = 1/2$ . Indeed, the total share of land devoted to crops at the symmetric equilibrium is the same, so we have  $B(z) + B^*(z) = 2$  for all  $z$ , but the relative importance of Home is decreasing with  $z$  (from  $B(\underline{z}) = 2$  to  $B(\bar{z}) = 0$ ), along with its relative productivity. It is only for  $z = 1/2$  that acreage and productivity are the same in both countries. Away from this crop, production becomes increasingly specialized and volatility increases. Volatility is the highest for all crops that are produced by only one country: using (38) with  $B(z) = 2$ , we get

$$v(\tilde{y}(z)) = \left( \frac{t(1 + 2t\kappa) - \mu}{2\mu N} \right)^{1/2}$$

for all  $z \in [0, \underline{z}] \cup [\bar{z}, 1]$ .

Figures 1 and 2 illustrate these findings. The solid curves with the marks depict the crop average price in each case as indicated (depicted are Home's autarky prices). The corresponding dashed curves depict the approximate values of  $p_d(z)$  and  $p_u(z)$ . The vertical distance between these curves defines a confidence interval for a crop price at a confidence level equal to 95%. Compared to autarky, the non-strategic average prices are larger for almost all crops, and the confidence intervals are very large. This is due to the tightening of the pesticide regulations mentioned above. The strategic effects that loosen these regulations induce lower average prices and confidence intervals. Biodiversity effects are reflected in Fig. 2 by strategic and non-strategic average price curves that encompass a flat portion around  $z = 1/2$  which corresponds to the mix-production range. Confidence intervals over these ranges are smaller the closer the crop is to  $z = 1/2$ .

Tables 1 and 2 assess the way the biodiversity effects and potential yield differentials affect the trade pattern, the environmental tax policy<sup>21</sup> and food price average level and volatility.<sup>22</sup> Table 1 summarizes the impacts due to the biodiversity effects: the larger  $\kappa$  is, the larger the

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<sup>21</sup>The tables report the value of  $\tau/w$ , which follows the trend of  $\tau$ , the environmental tax. Indeed, at equilibrium,  $w$ , the sum of the wage and the rental price of land, depends only on  $b$  (share of expenditures in the industrial good) and  $l$  (relative share of the industrial labor force) and is the same in autarky and under trade, since we are in a symmetric case, with  $q = w/(w + w^*) = 1/2$ .

<sup>22</sup>Food price levels are reported through a food price index given by  $\int_0^1 p(z)y(z)dz / \int_0^1 y(z)dz$ . The way it is approximated is detailed in the appendix. A food price index of 0.77 means that the average price of agricultural goods is 0.77% of the average price of industrial goods.



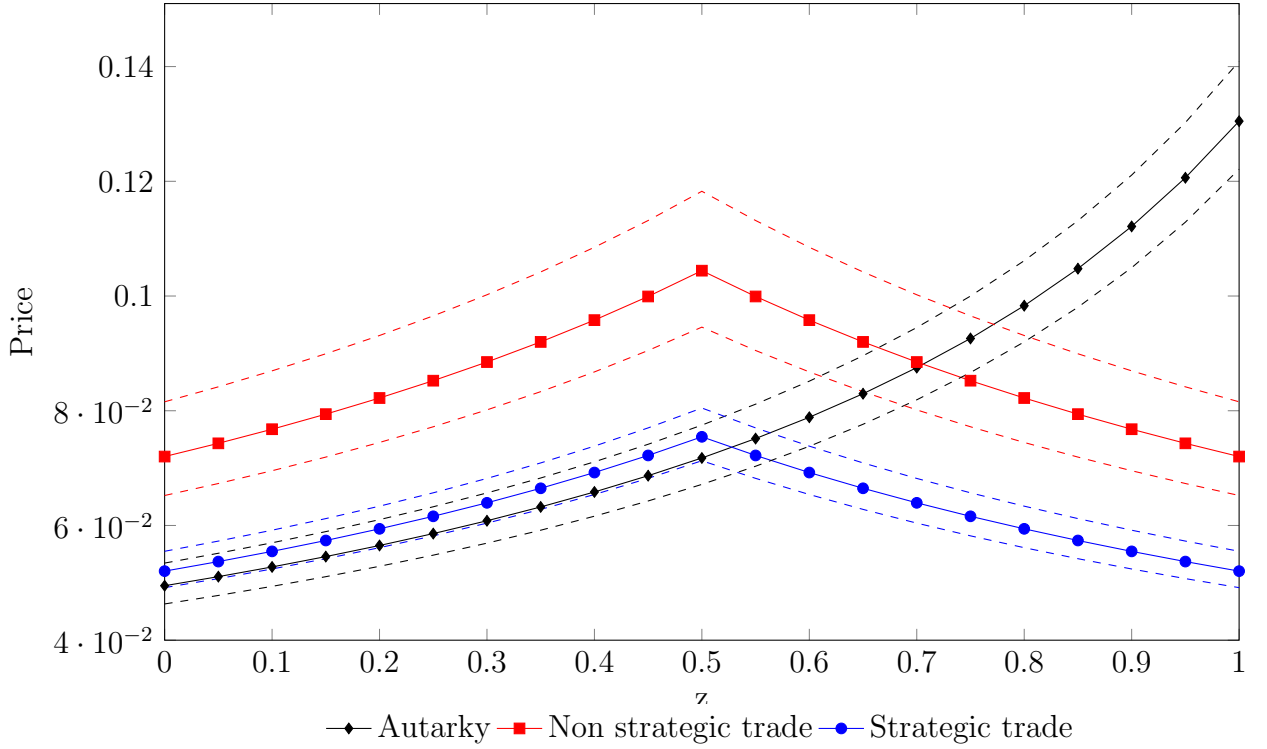


Figure 1: Average prices and price volatility (confidence interval at 95% confidence level) without biodiversity effects.  $\kappa = 0$ ,  $N=100$ ,  $h = 10^{-3}$ ,  $b = 0.8$ ,  $\ell = 20$ ,  $m = 0.45$ ,  $\mu = 1$ ,  $a_{cst} = 75$ .

negative impact of the cultivation intensity on the survival probability of plots. Biodiversity effects play against the specialization induced by trade: the range of crops produced by both countries increases when  $\kappa$  rises. Since biodiversity effects impede production, prices increase with  $\kappa$  as shown by the trend in the food price index reported in table 1. The larger  $\kappa$  is, the more effective pesticides are in reducing the negative externality due to the cultivation intensity and, therefore, the lower the environmental tax. Nevertheless, pesticides do not allow farmers to eradicate the biodiversity effect. Thus, even if more pesticides are applied when  $\kappa$  increases, the food price volatility increases.

Table 2 describes the impact of the potential crop yield differentials: the larger  $m$  is, the larger the difference in the potential yield of each crop (away from  $z = 1/2$ ), and the greater the difference in comparative advantages of the two countries. The range of crops that are produced by both countries under free trade ( $\bar{z} - \underline{z}$ ) decreases with  $m$ . As shown in table 2,  $m$

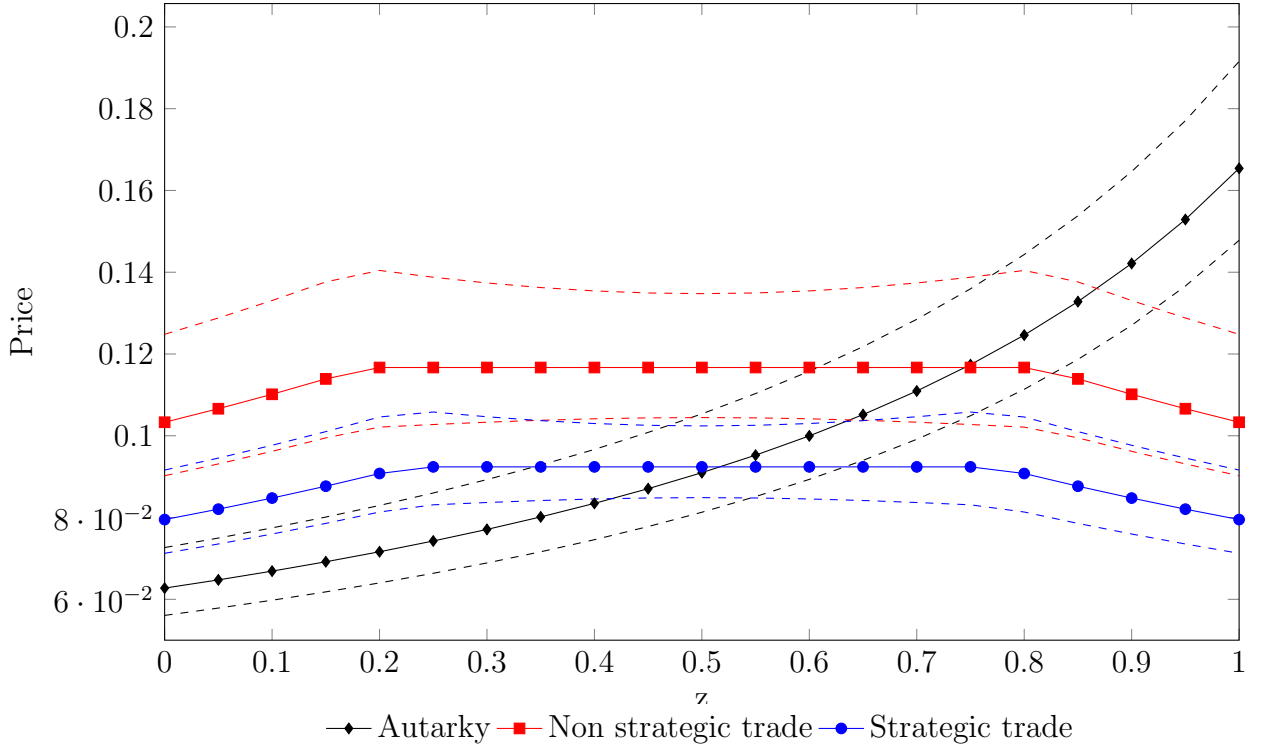


Figure 2: Average prices and price volatility (confidence interval at 95% confidence level) with biodiversity effects.  $\kappa = 0.3$ ,  $N=100$ ,  $h = 10^{-3}$ ,  $b = 0.8$ ,  $\ell = 20$ ,  $m = 0.45$ ,  $\mu = 1$ ,  $a_{cst} = 75$ .

has different effects on the environmental tax depending on whether trade is strategic or not. Under non strategic trade, the impact of the environmental policy on consumer welfare goes through the crop prices only. Therefore, the larger  $m$  is, the larger the specialization and the lower the tax on pesticides. Under strategic trade, the use of pesticides may have a significant impact on the market share of the country, particularly when comparative advantages are sufficiently close. This is reflected in table 2: the smaller  $m$  is, the lower the environmental tax. The way the food price index varies with  $m$  also differs between the non strategic and the strategic case. Two effects play on the quantities produced: on the one hand, when  $m$  raises, the productivity increases and, on the other hand, when the environmental tax increases, the quantities produced decrease. In the non strategic case, when  $m$  increases, the tax decreases. Then, the two effects go in the same direction, the quantities produced increase and prices decline. Under strategic trade,  $m$  and the environmental tax both increase, their effects are

Table 1: Sensitivity analysis on the parameter  $\kappa$ 

Variables	Values of $\kappa$				
	0.1	0.3	0.5	0.7	0.9
Autarky					
$\tau/w$	45.46	40.15	37.07	35.05	33.61
Food price index	0.77	0.90	1.03	1.16	1.30
$v(\tilde{p}(1/2))$	8.62	10.10	11.39	12.53	13.57
Non strategic trade					
$\tau/w$	83.29	73.23	69.03	66.49	64.68
Food price index	0.86	1.06	1.26	1.45	1.63
$v(\tilde{p}(1/2))$	8.00	8.84	9.76	10.62	11.42
$v(\tilde{p}(\underline{z}))$	8.82	10.61	12.19	13.55	14.75
$\bar{z} - \underline{z}$	20.66	46.96	64.70	77.69	87.65
Strategic trade					
$\tau/w$	52.51	44.83	40.99	38.53	36.76
Food price index	0.67	0.85	1.01	1.16	1.30
$v(\tilde{p}(1/2))$	6.39	7.37	8.28	9.10	9.85
$v(\tilde{p}(\underline{z}))$	7.07	8.91	10.42	11.70	12.84
$\bar{z} - \underline{z}$	17.16	41.51	58.71	71.64	81.76

*Note:* All variables are displayed in percentage values. The values of parameters are  $m = 0.6$ ,  $N=100$ ,  $h = 10^{-3}$ ,  $b = 0.8$ ,  $l = 20$ ,  $\mu = 0.7$ ,  $a_{cst} = 770$ .

countervailing on the quantities produced. The impact of the increase in the productivity prevails and the food price index decreases, even if the environmental tax is raised. Finally, in both strategic and non strategic cases, price volatility increases when the use of pesticides declines, i.e. when the environmental tax raises.

## 5 Fertilizers

Our focus being on biodiversity effects in agricultural production, we have so far not discussed the impact of trade on the use of fertilizers. However, because they have considerable effects on both crop yields and on the environment (and human health), changes in the openness of countries to trade are likely to impact the way their use is regulated.<sup>23</sup> We may thus expect

<sup>23</sup>Commercial fertilizers are responsible for 30% to 50% of crop yields (Stewart et al., 2005). Sutton et al. (2011a,b) find that half of the nitrogen added to farm fields ends up polluting water or air. Excess

Table 2: Sensitivity analysis on the parameter  $m$ 

Variables	Values of $m$			
	0.2	0.4	0.6	0.8
Autarky				
$\tau/w$	45.46	45.46	45.46	45.46
Food price index	0.77	0.77	0.77	0.77
$v(\tilde{p}(1/2))$	8.62	8.62	8.62	8.62
Non strategic trade				
$\tau/w$	86.02	83.94	83.29	82.97
Food price index	1.02	0.93	0.86	0.80
$v(\tilde{p}(1/2))$	8.19	8.05	8.00	7.98
$v(\tilde{p}(\underline{z}))$	9.02	8.87	8.82	8.80
$\bar{z} - \underline{z}$	63.23	31.13	20.66	15.46
Strategic trade				
$\tau/w$	46.82	49.83	52.51	54.81
Food price index	0.75	0.71	0.67	0.63
$v(\tilde{p}(1/2))$	6.18	6.29	6.39	6.49
$v(\tilde{p}(\underline{z}))$	6.84	6.96	7.07	7.17
$\bar{z} - \underline{z}$	50.19	25.43	17.16	13.01

*Note:* All variables are displayed in percentage values. The values of parameters are  $\kappa = 0.1$ ,  $N=100$ ,  $h = 10^{-3}$ ,  $b = 0.8$ ,  $l = 20$ ,  $\mu = 0.7$ ,  $a_{cst} = 770$ .

that crop price volatility is also affected through this channel. It is possible to analyze these changes by considering that crop  $z$ 's potential yield  $\bar{a}(z)$  is the result of the intrinsic quality of land and the quantity of fertilizers spread on the field,  $g(z)$ . Denoting by  $a_0(z)$  the potential crop  $z$  yield absent any treatment, we have  $\bar{a}(z) = a_0(z)f(g(z))$  with  $f(0) = 1$ ,  $f'(g) > 0$  and  $f''(g) < 0$ . Total use of fertilizers, given by  $G = N \int_0^1 B(z)g(z)dz$ , has a negative impact on consumer welfare due to environmental damages. As pesticides, fertilizers have a direct positive impact on crop yields, but unlike pesticides, their productive impact is limited to the field they are spread on. Hence, the trade-off that defines the fertilizer policy is similar to the one of the pesticide regulation without biodiversity effects. While in autarky domestic

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of nitrogen and phosphorus in freshwater increases cancer risk and creates aquatic and marine dead zones through eutrophication. In the air, nitrates contribute to ozone generation which causes respiratory and cardiovascular diseases. Sutton et al. (2011a) estimates that in the European Union the benefits of nitrogen for agriculture through the increase in yields amount to 25 to 130 billion euros per year and that they cause between 70 and 320 billion euros per year in damage.

consumers bear all the costs and reap all the benefits of the fertilizers used by their fellow farmers, this is no longer the case in free trade: they benefit from the crops produced abroad and share the advantages of a productive national sector with foreign consumers. As a result, restrictions on fertilizers are tighter under free trade than under autarky, with the same caveat as for pesticides: governments may use the fertilizer policy strategically. How lenient they are depends on the impact of fertilizers on relative yields: the more responsive is the relative yields function, i.e. the larger  $f'(g)$ , the lower the restrictions.

## 6 Conclusion

We have shown that biotic risk factors such as pests that affect the productivity of farming create biodiversity effects that modify standard results of trade models. An explicit account of their effects on production allows us to clarify the distribution of idiosyncratic risks affecting farming which depends on the countries' openness to trade. These production shocks translate into the price distribution and are impacted by environmental policies. Of course, these productive shocks are not the only ones affecting food prices, but they are an additional factor in their distributions that may explain their greater volatility compared to manufacture prices.

While these effects are analytically apparent within the standard two-country Ricardian model, an extension to a more encompassing setup involving a larger number of countries as permitted by Eaton & Kortum (2002) and applied to agricultural trade by Costinot & Donaldson (2012) and Costinot et al. (2012) is necessary to investigate their scope statistically. These studies incorporate a stochastic component to determine the pattern of trade but it is not related to the production process and somehow arbitrary. Our analysis offers an interesting route to ground these approaches at least in the case of agricultural products. Introducing these biodiversity effects in these approaches should allow for a better assessment of the importance to trade costs in determining the pattern of trade.

## References

- Beketov, M. A., Kefford, B. J., Schafer, R. B., & Liess, M. (2013). Pesticides reduce regional biodiversity of stream invertebrates. *Proceedings of the National Academy of Sciences*, 110(27), 11039–11043.
- Brander, J. A. & Spencer, B. J. (1985). Export subsidies and international market share rivalry. *Journal of International Economics*, 18(1-2), 83–100.
- Costinot, A. & Donaldson, D. (2012). Ricardo’s theory of comparative advantage: Old idea, new evidence. *American Economic Review*, 102(3), 453–58.
- Costinot, A., Donaldson, D., & Komunjer, I. (2012). What goods do countries trade? a quantitative exploration of ricardo’s ideas. *The Review of Economic Studies*, 79(2), 581–608.
- Di Falco, S. & Chavas, J. (2006). Crop genetic diversity, farm productivity and the management of environmental risk in rainfed agriculture. *European Review of Agricultural Economics*, 33(3), 289–314.
- Di Falco, S. & Perrings, C. (2005). Crop biodiversity, risk management and the implications of agricultural assistance. *Ecological Economics*, 55(4), 459–466.
- Dornbusch, R., Fischer, S., & Samuelson, P. A. (1977). Comparative advantage, trade, and payments in a ricardian model with a continuum of goods. *The American Economic Review*, 67(5), pp. 823–839.
- Drakare, S., Lennon, J. J., & Hillebrand, H. (2006). The imprint of the geographical, evolutionary and ecological context on species-area relationships. *Ecology Letters*, 9(2), 215–227.
- Eaton, J. & Kortum, S. (2002). Technology, geography, and trade. *Econometrica*, 70(5), 1741–1779.

- EC (2009). *EU Action on pesticides "Our food has become greener"*. Technical report, European Commission.
- ECP (2013). *Industry statistics*. Technical report, European Crop Protection.
- EEA (2010). *10 messages for 2010 – Agricultural ecosystems*. Technical report, European Environment Agency.
- Fernandez-Cornejo, J., Jans, S., & Smith, M. (1998). Issues in the economics of pesticide use in agriculture: a review of the empirical evidence. *Review of Agricultural Economics*, 20(2), 462–488.
- Garcia Martin, H. & Goldenfeld, N. (2006). On the origin and robustness of power-law species-area relationships in ecology. *Proceedings of the National Academy of Sciences*, 103(27), 10310–10315.
- Gilbert, C. L. & Morgan, C. W. (2010). Food price volatility. *Philosophical Transactions of the Royal Society B: Biological Sciences*, 365(1554), 3023–3034.
- Helpman, E. & Krugman, P. R. (1989). *Trade Policy and Market Structure*. Cambridge, MA.: MIT Press.
- Jacks, D. S., O'Rourke, K. H., & Williamson, J. G. (2011). Commodity price volatility and world market integration since 1700. *Review of Economics and Statistics*, 93(3), 800–813.
- Jiguet, F., Devictor, V., Julliard, R., & Couvet, D. (2012). French citizens monitoring ordinary birds provide tools for conservation and ecological sciences. *Acta Oecologica*, 44(0), 58–66.
- May, R. M. (2000). Species-area relations in tropical forests. *Science*, 290(5499), 2084–2086.
- Newbery, D. M. G. & Stiglitz, J. E. (1984). Pareto inferior trade. *The Review of Economic Studies*, 51(1), pp. 1–12.
- Oerke, E. (2006). Crop losses to pests. *Journal of Agricultural Science*, 144(1), 31.

- Pimentel, D. (2005). Environmental and economic costs of the application of pesticides primarily in the United States. *Environment, Development and Sustainability*.
- Plotkin, J. B., Potts, M. D., Yu, D. W., Bunyavejchewin, S., Condit, R., Foster, R., Hubbell, S., LaFrankie, J., Manokaran, N., Lee, H.-S., & et al. (2000). Predicting species diversity in tropical forests. *Proceedings of the National Academy of Sciences*, 97(20), 10850–10854.
- Savary, S., Willocquet, L., Elazegui, F. A., Castilla, N. P., & Teng, P. S. (2000). Rice pest constraints in tropical asia: Quantification of yield losses due to rice pests in a range of production situations. *Plant Disease*, 84(3), 357–369.
- Smale, M., Hartell, J., Heisey, P. W., & Senauer, B. (1998). The contribution of genetic resources and diversity to wheat production in the punjab of Pakistan. *American Journal of Agricultural Economics*, 80(3), 482–493.
- Stewart, W. M., Dibb, D. W., Johnston, A. E., & Smyth, T. J. (2005). The contribution of commercial fertilizer nutrients to food production. *Agronomy Journal*, 97(1), 1.
- Storch, D., Keil, P., & Jetz, W. (2012). Universal species – area and endemics – area relationships at continental scales. *Nature*, 488(7409), 78–81.
- Sutton, M., Howard, C., Erisman, J., Billen, G., Bleeker, A., Grennfelt, P., Grinsven, v. H., & Grizzetti, B., Eds. (2011a). *The European Nitrogen Assessment: Sources, Effects and Policy Perspectives*. Cambridge University Press.
- Sutton, M. A., Oenema, O., Erisman, J. W., Leip, A., van Grinsven, H., & Winiwarter, W. (2011b). Too much of a good thing. *Nature*, 472(7342), 159–161.
- Tilman, D. & Downing, J. A. (1994). Biodiversity and stability in grasslands. *Nature*, 367(6461), 363–365.
- Tilman, D., Polasky, S., & Lehman, C. (2005). Diversity, productivity and temporal stability in the economies of humans and nature. *Journal of Environmental Economics and Management*, 49(3), 405–426.



- Tilman, D., Wedin, D., & Knops, J. (1996). Productivity and sustainability influenced by biodiversity in grassland ecosystems. *Nature*, 379(6567), 718–720.
- Weitzman, M. L. (2000). Economic profitability versus ecological entropy. *The Quarterly Journal of Economics*, 115(1), pp. 237–263.
- Wright, B. D. (2011). The economics of grain price volatility. *Applied Economic Perspectives and Policy*, 33(1), 32–58.

# Appendix

## A Proof of Proposition 1

For given  $t, t^*, w, w^*$ , (12) and (13) define a system of two linear equations with two unknowns. Solving this system gives (16). By definition of threshold crops  $\underline{z}$  and  $\bar{z}$ , we must have  $B(z) = 0$  for all  $z \geq \bar{z}$  and  $B^*(z) = 0$  for all  $z \leq \underline{z}$ . This implies that we must have  $\phi(z) \geq 1/q$  or all  $z \geq \bar{z}$  and  $\phi^*(z) \geq 1/q^*$  for all  $z \leq \underline{z}$ . Differentiating (17) and its counterpart for Foreign, we get

$$\dot{\phi}(z) \equiv \frac{\phi'(z)}{\phi(z)} = \frac{A'(z)}{t^*/t + A(z)} - t^* \kappa_0 e^{-\theta(z)^2/2} \frac{\alpha'(z) - \theta'(z)\alpha(z)}{1 + t^* \alpha(z) \kappa(z)}$$

and

$$\dot{\phi}^*(z) = -\dot{A}(z) + \frac{A'(z)}{t^*/t + A(z)} - t \kappa_0 e^{-\theta(z)^2/2} \frac{\alpha'(z) - \theta'(z)\alpha(z)}{1 + t^* \alpha(z) \kappa(z)}.$$

Suppose  $\kappa_0 = 0$ : As  $\dot{A}(z) > 0$ , we have  $\dot{\phi}(z) = A'(z)/[t^*/t + A(z)] > 0$  and  $\dot{\phi}^*(z) = -\dot{A}(z)(t^*/t)/[t^*/t + A(z)] < 0$ . Both conditions are thus satisfied if  $\kappa_0$  is sufficiently small. They are also satisfied whatever the value of  $\kappa_0$  if  $\alpha(z)$  and  $\theta(z)$  are constant as supposed in the symmetric case considered in section 3. Eq. (14) and (15) are derived from  $\phi(\bar{z}) = 1/q$  and  $\phi^*(\underline{z}) = 1/q^*$  respectively. Using these equations, we obtain  $A(\bar{z})/A(\underline{z}) = [q^* + t^* \kappa(\bar{z}) \alpha(\bar{z})][q + t \kappa(\underline{z}) \alpha(\underline{z})]/(qq^*) > 1$ . As  $A(z)$  is increasing, we thus have  $\bar{z} > \underline{z}$ .

## B Proof of Lemma 1

Consider the crops produced more intensively by Home (symmetric results hold for Foreign). For all  $z < \underline{z}$ , it is obvious by setting  $t = t_A$  and  $q = 1/2$  in (21) that  $\bar{p}(z) > \bar{p}_A(z)$  given by (9). For Foreign, we have  $\bar{p}(z) < \bar{p}_A^*(z)$  iff

$$\frac{1 + t_A \kappa(z) \alpha(z)}{1 + 2t_A \kappa(z) \alpha(z)} > A(z).$$

As  $\phi^*(z) > 1/q^*$  for all  $z < \underline{z}$ , we obtain by setting  $t = t_A$  and  $q^* = 1/2$  that

$$\frac{1 + 1/A(z)}{1 + t_A \kappa(z) \alpha(z)} > 2$$

for all  $z < \underline{z}$ , which gives, re-arranging terms

$$A(z) < \frac{1}{1 + 2t_A \kappa(z) \alpha(z)} < \frac{1 + t_A \kappa(z) \alpha(z)}{1 + 2t_A \kappa(z) \alpha(z)}$$

hence  $\bar{p}(z) < \bar{p}_A^*(z)$  for all  $z < \underline{z}$ . For crops  $z$  in  $[\underline{z}, \bar{z}]$ , we obtain using (20) that  $\bar{p}(z) < \bar{p}_A(z)$  iff  $2/[1 + A(z)] < 1$ , hence the result.

## C Proof of Proposition 3

Using (21) which simplifies to

$$\bar{p}_s(z) = \frac{2(\ell - 1)(1 - b)tq}{ba(z)}$$

and denoting by  $\varpi$  the Lagrange multiplier associated with the constraint, the first-order condition with respect to  $t$  gives

$$-(1 - b)q + \frac{Nh}{\sqrt{2 \ln t}} - \varpi \frac{1}{\xi'(z_s)} \frac{t^*}{t} = 0$$

and the one with respect to  $q$

$$\varpi = \frac{2(1 - b)}{b + 2q(1 - b)}.$$

Plugging the latter expression into the former and rearranging terms gives (29). At a symmetric equilibrium, using  $\xi'(z_s) = A'(1/2) + 4$  and  $\tau_A/w_A = Nh/(1 - b)$ , we get

$$\frac{\tau}{w} = \frac{\tau_A}{w_A} \left[ 1 + \frac{A'(1/2)}{A'(1/2) + 8} \right]$$

where  $w = w_A$ . We thus have

$$\tau = \tau_A \left[ 1 + \frac{A'(1/2)}{A'(1/2) + 8} \right].$$

Denoting  $M \equiv A'(1/2)$ , we get  $\lim_{M \rightarrow 0} \tau = \tau_A$ ,  $\lim_{M \rightarrow +\infty} \tau = 2\tau_A$  and

$$\frac{d\tau}{dM} = \frac{8\tau_A}{(M+8)^2} > 0.$$

## D Proof of Proposition 4

Differentiating (26) with respect to  $t$ , we obtain the following FOC

$$\frac{\partial V}{\partial t} = -(1-b) \left[ \bar{z} \frac{q+2t\kappa}{t(q+t\kappa)} + \frac{(\bar{z}-\underline{z})(q^*+t^*\kappa)}{q(t^*-t)+t(1+t^*\kappa)} - \int_{\underline{z}}^{\bar{z}} \frac{A(z)/t^*-t^*/t^2}{A(z)t/t^*+t^*/t} dz \right] - h \frac{dZ}{dt}.$$

At a symmetric equilibrium, using  $(A(z)-1)/(A(z)+1) = m(2z-1)$  and integrating gives (34). Using (33) in (34) and collecting terms, we arrive at

$$\frac{\partial V}{\partial t} = \frac{Nh}{t\sqrt{2\ln t}} - (1-b) \left[ \frac{1}{2t} + \kappa \frac{2m(1+t\kappa)^2 - t\kappa}{2m(1+2t\kappa)(1+t\kappa)^2} \right].$$

Denote by  $t_0$  the optimal tax when there is no cross-externality effects, i.e.  $\kappa = 0$ . It verifies (28) where  $q = 1/2$ . We have

$$\left. \frac{\partial V}{\partial t} \right|_{t=t_0} = -(1-b)\kappa \frac{2m(1+t_0\kappa)^2 - t_0\kappa}{2m(1+2t_0\kappa)(1+t_0\kappa)^2}$$

which is positive if

$$m \leq \frac{t_0\kappa}{2(1+t_0\kappa)^2} \leq 1/8.$$

We also have

$$\begin{aligned}
\left. \frac{1}{1-b} \frac{\partial V}{\partial t} \right|_{t=t_A} &= \frac{1}{2t_A} + \frac{\kappa}{1+t_A\kappa} - \kappa \frac{2m(1+t_A\kappa)^2 - t_A\kappa}{2m(1+2t_A\kappa)(1+t_A\kappa)^2} \\
&= \frac{1}{2t_A} + \frac{t_A\kappa^2}{1+t_A\kappa} \frac{2m(1+t_A\kappa) + 1}{2m(1+2t_A\kappa)(1+t_A\kappa)} > 0
\end{aligned} \tag{46}$$

hence  $t > t_A$  at the optimum.

## E Proof of Proposition 5

At a symmetric equilibrium (35) simplifies to

$$\frac{\partial V}{\partial q} = 2(1-b) + 2(1-b) \frac{1 - (\bar{z} + \underline{z})}{1 + 2t\kappa}$$

where  $\bar{z} + \underline{z} = 1$ , hence  $\partial V / \partial q = 2(1-b)$ . We also obtain using (27) that at a symmetric equilibrium

$$\frac{dq}{dt} = - \frac{\frac{2(1+t\kappa)}{t} \int_{\underline{z}}^{\bar{z}} \frac{A(z)}{[A(z)+1]^2} dz - \frac{1}{2t} \int_{\underline{z}}^{\bar{z}} \frac{A(z)}{A(z)+1} dz}{t\kappa + \bar{z} - \underline{z}}.$$

Denoting  $m_0 = (m+1)/2$ , using

$$\begin{aligned}
\int_{\underline{z}}^{\bar{z}} \frac{A(z)}{[A(z)+1]^2} dz &= \int_{\underline{z}}^{\bar{z}} [m_0 - mz - (m_0 - mz)^2] dz \\
&= (\bar{z} - \underline{z})(m_0 - m/2) + \frac{(m_0 - m\bar{z})^3 - (m_0 - m\underline{z})^3}{3m} \\
&= \frac{t\kappa}{2m(1+t\kappa)} \frac{2(1+t\kappa)^2 + 1 + 2t\kappa}{6(1+t\kappa)^2}
\end{aligned}$$

and

$$\int_{\underline{z}}^{\bar{z}} \frac{A(z)}{A(z)+1} dz = \int_{\underline{z}}^{\bar{z}} (1 - m_0 + mz) dz = (\bar{z} - \underline{z})(1 - m_0 + m/2) = \frac{t\kappa}{2m(1+t\kappa)}$$

yields (36). Differentiating, we get

$$\frac{dq^2}{dt d\kappa} = -\frac{(4-m)(1+t\kappa)^2 + 4m(1+t\kappa) + 2}{12(1+t\kappa)^2[m(1+t\kappa) + 1]^2} < 0.$$

Using (46) we obtain

$$\frac{1}{1-b} \frac{dV}{dt} \Big|_{t=t_A} = \frac{1}{2t_A} + \frac{t_A \kappa^2 [2m(1+t_A \kappa) + 1]}{2m(1+2t_A \kappa)(1+t_A \kappa)^2} - \frac{3 + 9t_A \kappa + 4(t_A \kappa)^2}{6t_A(1+t_A \kappa)[m(1+t_A \kappa) + 1]} \quad (47)$$

where the second term diverges when  $m$  tends to 0 and decreases with  $m$  while the last term increases with  $m$  (decreases in absolute value). Assuming that  $m$  is large enough and  $\kappa \approx 0$ , we have

$$\begin{aligned} \frac{1}{1-b} \frac{dV}{dt} \Big|_{t=t_A} &= \frac{1}{2t_A} - \frac{1 + 3t_A \kappa}{2t_A[m(1+2t_A \kappa) + 1 + t_A \kappa]} + o(\kappa) \\ &= \frac{1}{2t_A} \left[ \frac{m(1+2t_A \kappa) - 2t_A \kappa}{m(1+2t_A \kappa) + (1+t_A \kappa)} \right] + o(\kappa) \end{aligned}$$

where the bracketed term is negative if

$$m < \bar{m} \equiv \frac{2t_A \kappa}{1 + 2t_A \kappa}.$$

However, as the second term of (47) diverges when  $m$  approaches 0, we cannot neglect this term unless  $m$  is above some threshold  $\underline{m} > t_A \kappa / (1 + t_A \kappa)$ . Indeed, in the case where  $m = t_A \kappa / (1 + t_A \kappa)$ , we have

$$\begin{aligned} \frac{1}{1-b} \frac{dV}{dt} \Big|_{t=t_A} &= \frac{1}{2t_A} + \frac{\kappa}{2(1+t_A \kappa)} - \frac{3 + 9t_A \kappa + 4(t_A \kappa)^2}{6t_A(1+t_A \kappa)^2} \\ &= \frac{(t_A \kappa)^2}{3t_A(1+t_A \kappa)^2} > 0. \end{aligned}$$

Hence, assuming  $\kappa$  is small enough, cases where the environmental tax at equilibrium is

lower than under autarky correspond to  $m \in [\underline{m}, \bar{m}]$  where

$$\bar{m} - \underline{m} < \frac{2t_A\kappa}{1 + 2t_A\kappa} - \frac{t_A\kappa}{1 + t_A\kappa} = \frac{t_A\kappa}{(1 + t_A\kappa)(1 + 2t_A\kappa)} \leq \frac{\sqrt{2}}{4 + 3\sqrt{2}} \approx 0.17.$$

## F Proof of Proposition 6

A second-order approximation gives

$$\begin{aligned} \tilde{p}(z) &= \frac{\alpha(z)(1-b)LR}{\tilde{y}(z)} \approx \frac{\alpha(z)(1-b)LR}{y(z)} \left[ 1 - \frac{\tilde{y}(z) - y(z)}{y(z)} + \left( \frac{\tilde{y}(z) - y(z)}{y(z)} \right)^2 \right] \\ &= \bar{p}(z) \left[ 2 - \frac{\tilde{y}(z)}{y(z)} + \left( \frac{\tilde{y}(z) - y(z)}{y(z)} \right)^2 \right] \end{aligned}$$

and thus

$$p(z) \approx \bar{p}(z) \left[ 1 + E \left( \frac{\tilde{y}(z) - y(z)}{y(z)} \right)^2 \right] = \bar{p}(z)[1 + v(\tilde{y}(z))^2].$$

A first order approximation yields

$$\frac{E[(\tilde{p}(z) - \bar{p}(z))^2]^{1/2}}{\bar{p}(z)} \approx E \left[ \left( 1 - \frac{\tilde{y}(z)}{y(z)} \right)^2 \right]^{1/2} = v(\tilde{y}(z)),$$

which gives

$$\begin{aligned} \sigma(\tilde{p}(z)) &\approx E [\tilde{p}(z) - \bar{p}(z) - \bar{p}(z)v(\tilde{y}(z))^2]^{1/2} = (E [(\tilde{p}(z) - \bar{p}(z))^2] - \bar{p}(z)^2 v(\tilde{y}(z))^4)^{1/2} \\ &= \bar{p}(z)v(\tilde{y}(z))(1 - v(\tilde{y}(z))^2)^{1/2}. \end{aligned}$$

From  $p_u^\gamma(z) = p(z) + s_u^\gamma \sigma(\tilde{p}(z))$  we get

$$\begin{aligned}
s_u^\gamma &\approx \frac{1}{\sigma(\tilde{p}(z))} \left( \frac{\alpha(z)(1-b)LR}{y(z) - s_\gamma \sigma(\tilde{y}(z))} - \bar{p}(z)(1 + v(\tilde{y}(z))^2) \right) \\
&= \frac{\bar{p}(z)}{\sigma(\tilde{p}(z))} \frac{y(z)(1 + v(\tilde{y}(z))^2) - y(z) + s_\gamma \sigma(\tilde{y}(z))}{y(z) - s_\gamma \sigma(\tilde{y}(z))} \\
&\approx \frac{y(z)v(\tilde{y}(z))^2 + s_\gamma \sigma(\tilde{y}(z))}{v(\tilde{y}(z))(1 - v(\tilde{y}(z))^2)^{1/2}(y(z) - s_\gamma \sigma(\tilde{y}(z)))} \\
&= \frac{v(\tilde{y}(z)) + s_\gamma}{(1 - v(\tilde{y}(z))^2)^{1/2}(1 - s_\gamma v(\tilde{y}(z)))}
\end{aligned}$$

which gives (43). Similar derivations for  $p_d^\gamma(z) = E[\tilde{p}(z)] - s_d^\gamma \sigma(\tilde{p}(z))$  yield (42) and (44).

## G Food Price Index

The food price index is defined as

$$F = \int_0^1 y(z)p(z)dz \Big/ \int_0^1 y(z)dz$$

where  $p(z)$  is approximated by (40). As we also have  $\bar{p}(z)y(z) = wNB(z)$ , we get  $p(z)y(z) \approx wNB(z)[1 + v(\tilde{y}(z))^2]$ . In autarky,  $w = (\ell - 1)(1 - b)/b$ . Using  $B(z) = 1$ ,  $\theta(z) = \theta$ , which implies  $\mu(z) = \mu$ , and  $y(z) = a(z)N/[t(1 + t\kappa)]$  gives

$$\int_0^1 p(z)y(z)dz = \frac{N(\ell - 1)(1 - b)}{b} \left[ 1 + \frac{t(1 + t\kappa) - \mu}{N\mu} \right].$$

Using  $a(z) = \mu_0 e^{\theta^2/2} [1 - m(2z - 1)]$  yields

$$\int_0^1 y(z)dz = \frac{N}{t(1 + t\kappa)} \int_0^1 \mu_0 e^{\theta^2/2} [1 - m(2z - 1)]dz = \frac{N\mu_0 e^{\theta^2/2}}{t(1 + t\kappa)}$$

and thus

$$F_A = \frac{(\ell - 1)(1 - b)t(1 + t\kappa)}{b\mu_0 e^{\theta^2/2}} \left[ 1 + \frac{t(1 + t\kappa) - \mu}{N\mu} \right].$$

Under free trade, we have  $\bar{p}(z)y(z) = 2N(\ell - 1)(1 - b)/b$ .  $B(z) = 2$  for crops produced



by only one country and we get, using  $\underline{z} + \bar{z} = 1$ ,

$$\int_0^{\underline{z}} p_s(z)y(z)dz + \int_{\bar{z}}^1 p_s(z)y^*(z)dz = \frac{4\underline{z}N(\ell-1)(1-b)}{b} \left[ 1 + \frac{t(1+2t\kappa) - \mu}{2\mu N} \right].$$

For crops produced by both countries we obtain

$$\begin{aligned} \int_{\underline{z}}^{\bar{z}} p_m(z)y_W(z)dz &= \int_{\underline{z}}^{\bar{z}} \frac{2N(\ell-1)(1-b)}{b} \left[ 1 + \frac{(1+t\kappa)(1+2t\kappa) - \mu\kappa}{2\mu N\kappa} - \frac{2(1+t\kappa)^2}{\mu N\kappa} \frac{A(z)}{[1+A(z)]^2} \right] dz \\ &= \frac{2N(\ell-1)(1-b)}{b} \left\{ \left[ 1 + \frac{(1+t\kappa)(1+2t\kappa) - \mu\kappa}{2\mu N\kappa} \right] (\bar{z} - \underline{z}) - \frac{t[2(1+t\kappa)^2 + 1 + 2t\kappa]}{\mu N m 6(1+t\kappa)} \right\}. \end{aligned}$$

The corresponding quantities are given by

$$\begin{aligned} \int_0^{\underline{z}} y(z)dz + \int_{\bar{z}}^1 y(z)dz &= \frac{2N}{t(1+2t\kappa)} \left\{ \int_0^{\underline{z}} a(z)dz + \int_{\bar{z}}^1 a^*(z)dz \right\} \\ &= \frac{2N\mu_0 e^{\theta^2/2}}{t(1+2t\kappa)} [\underline{z}(m+1 - m\underline{z}) + 1 - \bar{z}(1 + m\bar{z} - m)] \end{aligned}$$

and

$$\int_{\underline{z}}^{\bar{z}} y_W(z)dz = \frac{N}{t(1+t\kappa)} \int_{\underline{z}}^{\bar{z}} [a^*(z) + a(z)]dz = \frac{2N\mu_0 e^{\theta^2/2}(\bar{z} - \underline{z})}{t(1+t\kappa)} = \frac{2N\mu_0 e^{\theta^2/2}\kappa}{m(1+t\kappa)^2}.$$